6.3. Trig Equations – Part II.

Monday, April 22, 2019   10:41 AM

Equations with trig functions of multiples of angles.

E.g. Solve for θ in $[0°, 360°)$: $\cos(\boxed{2\theta}) = \frac{\sqrt{3}}{2}$.

angle 2θ

$0° \le \theta < 360° \to 0° \le 2\theta < 720°$

$\to$ Solve the equation $\cos(2\theta) = \frac{\sqrt{3}}{2}$ for 2θ in $[0°, 720°)$

Then once we get 2θ, we can solve for θ by itself.

$\cos(\boxed{2\theta}) = \frac{\sqrt{3}}{2}$

$\to \boxed{2\theta} = \boxed{30°, 330°, 390°, 690°}$ in $[0°, 720°)$

$2\theta = 30°$; $2\theta = 330°$, $2\theta = 390°$; $2\theta = 690°$

$\boxed{\theta = 15° \ ; \ \theta = 165° \ ; \ \theta = 195° \ ; \ \theta = 345°}$

Ex. 1 Solve for x in $[0°, 360°)$: $3\tan(3x) = \sqrt{3}$

Ex. 2. Solve for x in $[0, 2\pi)$: $\sqrt{2}\sin(3x) - 1 = 0$

Sol: (1) $0° \le x < 360° \to 0° \le 3x < 1080°$

$3\tan(3x) = \sqrt{3} \to \tan(3x) = \frac{\sqrt{3}}{3}$

Monday, April 22, 2019 11:11 AM

$3x = 30°, 210°, 390°, 570°, 750°, 930°$

$\to \boxed{x = 10°, 70°, 130°, 190°, 250°, 310°}$

(2) $0 \le x < 2\pi \to 0 \le 3x < 6\pi.$

$$\sin(3x) = \frac{1}{\sqrt{2}} = \frac{\sqrt{2}}{2}$$

$$3x = \frac{\pi}{4}, \frac{3\pi}{4}, \frac{9\pi}{4}, \frac{11\pi}{4}, \frac{17\pi}{4}, \frac{19\pi}{4}$$

$$\boxed{x = \frac{\pi}{12}, \underbrace{\frac{3\pi}{12}}_{\frac{\pi}{4}}, \underbrace{\frac{9\pi}{12}}_{\frac{3\pi}{4}}, \frac{11\pi}{12}, \frac{17\pi}{12}, \frac{19\pi}{12}}$$

Ex. $\cos\left(\frac{x}{2}\right) = \sqrt{2} - \cos\left(\frac{x}{2}\right)$. Solve for x in $[0°, 360°)$

Ex. $2\sqrt{3}\sin\left(\frac{x}{2}\right) = 3$. Solve for x in $[0, 2\pi)$

Sol: (1) $0° \le x < 360° \to 0° \le x < 180°$

$$2\cos\left(\frac{x}{2}\right) = \sqrt{2} \to \cos\left(\frac{x}{2}\right) = \frac{\sqrt{2}}{2}$$

$$\to \frac{x}{2} = 45° \to \boxed{x = 90°}$$

Monday, April 22, 2019 11:31 AM

(2) $0° \leq x < 2\pi \rightarrow 0° \leq \frac{x}{2} < \pi$

$$\sin\left(\frac{x}{2}\right) = \frac{3}{2\sqrt{3}} = \frac{\sqrt{3}}{2}$$

$$\frac{x}{2} = \frac{\pi}{3} ; \frac{2\pi}{3} \rightarrow \boxed{x = \frac{2\pi}{3}, \frac{4\pi}{3}}$$

Use the Double angle identities.

E.g. $\sin(2\theta) = \sin\theta$. Solve for θ in $[0°, 360°)$

$$2\sin(\theta)\cos(\theta) = \sin(\theta)$$ (Double Angle Identity for Sine)

$$\rightarrow 2\sin(\theta)\cos(\theta) - \sin(\theta) = 0$$

$$\rightarrow \sin(\theta)\left[2\cos(\theta) - 1\right] = 0$$

$\sin(\theta) = 0$ or $\cos(\theta) = \frac{1}{2}$

$\boxed{\theta = 0°, 180°}$ $\boxed{\theta = 60°, 300°}$

Ex. $\cos(2x) = 1 - \sin(x)$. Solve for x in $[0, 2\pi)$

Double angle identity

$$\cancel{1} - 2\sin^2(x) = \cancel{1} - \sin(x)$$

$$0 = 2\sin^2 x - \sin x$$

$$0 = \sin x\,(2\sin x - 1)$$

$\sin x = 0$ or $\sin x = \frac{1}{2}$

$$\boxed{x = 0, \pi \qquad x = \frac{\pi}{6}, \frac{5\pi}{6}}$$