6.2 - Trig Equations Part I

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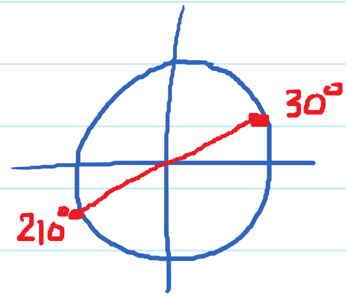
Linear Equations:

E.g. $3\tan\theta - \sqrt{3} = 0$

(a) Solve for θ in $[0°, 360°)$

$\rightarrow \quad 3\tan\theta = \sqrt{3}$

$\rightarrow \quad \tan\theta = \dfrac{\sqrt{3}}{3}$ (Isolated $\tan\theta$)



From unit circle, $\boxed{\theta = 30°, 210°}$

(b) Solve for all solutions:

$$\boxed{\begin{aligned} \theta &= 30° + n\cdot 360° \\ \theta &= 210° + n\cdot 360° \end{aligned}}$$

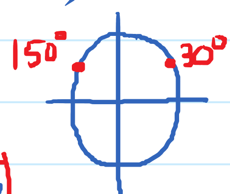
where $n = 0, 1, 2, 3 \ldots$
$-1, -2, -3, \ldots$

where $n = 0, \pm1, \pm2, \pm3, \ldots$

Ex. Solve the equation $2\sin x + 3 = 4$

(a) On $[0°, 360°)$

$\sin x = \dfrac{1}{2}$



$\boxed{x = 30°, 150°}$

(b) All solutions

$x = 30° + n\cdot 360°$

$x = 150° + n\cdot 360°$

where $n = 0, \pm1, \pm2, \pm3, \ldots$

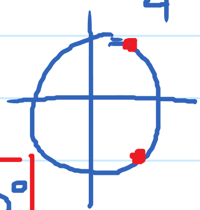
Root Extraction:

E.g. Solve equations: $4\cos^2 x - 1 = 0$

(a) On $[0°, 360°)$

$$\longrightarrow \cos^2 x = \frac{1}{4} \longrightarrow \cos x = \pm\sqrt{\frac{1}{4}} = \pm\frac{1}{2}$$

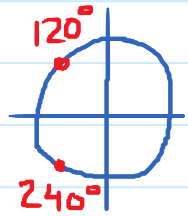
$\cos x = \frac{1}{2}$



$\boxed{x = 60°, 300°}$

or

$\cos x = -\frac{1}{2}$



$\boxed{x = 120°, 240°}$

(b) All solutions:

$$x = 60° + n \cdot 360° \;;\quad x = 300° + n \cdot 360°$$

$$x = 120° + n \cdot 360° \;;\quad x = 240° + n \cdot 360°$$

$$n = 0, \pm 1, \pm 2, \pm 3, \ldots$$

Ex. Solve $\tan^2 x + 3 = 0$

(a) On $[0, 2\pi)$

$\tan^2 x = -3$

$\tan x = \pm\sqrt{-3}$ non real

$\rightarrow$ No solutions

(b) All solutions

No solutions.

Quadratic by factoring.

E.g. $-2\sin^2 x = 3\sin x + 1.$

(a) Solve over $[0, 2\pi)$

$$0 = 2\sin^2 x + 3\sin x + 1$$

$\frac{1}{1} = \frac{2}{2}$ 2 multiply $\frac{1}{2}$ 3 add

$$0 = (\sin x + 1)(2\sin x + 1)$$

$\sin x + 1 = 0$ or $2\sin x + 1 = 0$

$\sin x = -1$ or $\sin x = -\frac{1}{2}$

$\boxed{x = \frac{3\pi}{2}}$ $\boxed{x = \frac{7\pi}{6}, \frac{11\pi}{6}}$.

(b) All solutions:

$$\boxed{x = \frac{3\pi}{2} + n\cdot 2\pi \ ; \ \frac{7\pi}{6} + n\cdot 2\pi, \ \frac{11\pi}{6} + n\cdot 2\pi}$$

where $n = 0, \pm 1, \pm 2, \pm 3, \ldots$

Ex. Solve the given equation on $[0, 2\pi)$ and then find all solutions

(a) $\tan^2 x + \tan x - 2 = 0$ | (b) $\sec^2\theta \tan\theta = 2\tan\theta$

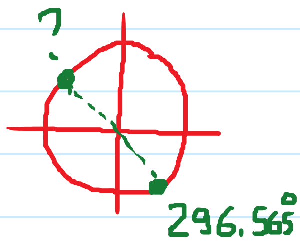
Sol:

(a) $\tan^2 x + \tan x - 2 = 0$

$(\tan x - 1)(\tan x + 2) = 0$

$\tan x - 1 = 0$ or $\tan x + 2 = 0$

$\tan x = 1$ or $\tan x = -2$



$x = \frac{\pi}{4}, \frac{5\pi}{4}$ $\qquad$ $x = -63.435°$

$\xrightarrow{+360°}$ $\boxed{x = 296.565°}$ → convert to rad

$x = 296.565° - 180°$

$\boxed{x = 116.565°}$ → convert to rad.

(b) All solutions: add $n \cdot 2\pi$ to each one of these solutions

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(b) $\sec^2\theta \tan\theta = 2\tan\theta$.

$\sec^2\theta \tan\theta - 2\tan\theta = 0$

$\tan\theta\left(\sec^2\theta - 2\right) = 0$

$\tan\theta = 0$

$\boxed{\theta = 0, \pi}$

or $\sec^2\theta - 2 = 0$

$\rightarrow \sec^2\theta = 2$

$\rightarrow \sec\theta = \pm\sqrt{2}$

$\rightarrow \cos\theta = \pm\frac{1}{\sqrt{2}} = \pm\frac{\sqrt{2}}{2}$

$\cos\theta = \frac{\sqrt{2}}{2}$; $\cos\theta = -\frac{\sqrt{2}}{2}$

$\boxed{\theta = \frac{\pi}{4}, \frac{7\pi}{4}}$ $\boxed{\theta = \frac{3\pi}{4}, \frac{5\pi}{4}}$

(b) All solutions: add $n \cdot 2\pi$ to each of these.

Use Identities and then factoring.

E.g. Solve on $[0°, 360°)$

(a) $2\sin\theta - 1 = \csc\theta$ (b) $5 + 5\tan^2\theta = 6\sec\theta$

(a) $2\sin\theta - 1 = \csc\theta$

$\sin\theta\left(2\sin\theta - 1\right) = \left(\frac{1}{\sin\theta}\right)\cdot\sin\theta$ (Multiply both sides by $\sin\theta$)

$2\sin^2\theta - \sin\theta = 1$

$2\sin^2\theta - \sin\theta - 1 = 0$

$(2\sin\theta + 1)(\sin\theta - 1) = 0$

$\sin\theta = -\frac{1}{2}$; $\sin\theta = 1$

$\rightarrow \boxed{\theta = 210^\circ, 330^\circ \quad ; \quad 90^\circ}$

(b) $5 + 5\tan^2\theta = 6\sec\theta$.

$5 + 5(\sec^2\theta - 1) = 6\sec\theta$

$\frac{1}{\cos\theta} = 0$

$\cancel{5} + 5\sec^2\theta \cancel{- 5} = 6\sec\theta$

$5\sec^2\theta - 6\sec\theta = 0$

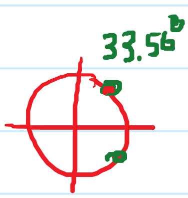
$\sec\theta(5\sec\theta - 6) = 0$

$\rightarrow \sec\theta = 0 \rightarrow$ no solution

$\rightarrow \sec\theta = \frac{6}{5}$

$\rightarrow \cos\theta = \frac{5}{6}$.

$\boxed{\theta = 33.56^\circ, 326.44^\circ}$



Square both sides and use identities.

E.g. $\tan(x) + \sqrt{3} = \sec(x)$ on $[0, 2\pi)$

→ Square both sides:

$$\boxed{\left(\tan(x) + \sqrt{3}\right)^2} = \underbrace{\sec^2(x)} \qquad (\tan x + \sqrt{3})(\tan x + \sqrt{3})$$

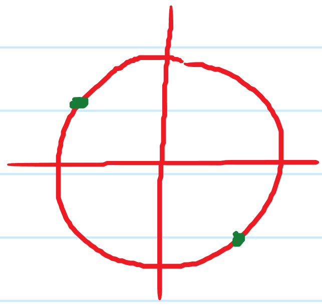
Identity

$$\cancel{\tan^2 x} + 2\sqrt{3}\tan x + \overset{2}{\cancel{3}} = \cancel{\tan^2(x)} + \cancel{1}$$

$$2\sqrt{3}\tan x + 2 = 0$$

$$\sqrt{3}\tan x + 1 = 0$$

$$\tan x = -\frac{1}{\sqrt{3}} = -\frac{\sqrt{3}}{3}.$$



$$x = \cancel{\frac{5\pi}{6}}, \frac{11\pi}{6}.$$

Check solutions: $\boxed{\cancel{x = \frac{5\pi}{6}}}$ → extraneous solution

$\boxed{x = \frac{11\pi}{6}}$ → solution

$$\tan(x) + \sqrt{3} = \sec(x)$$

$$-\frac{\sqrt{3}}{3} + \frac{3\sqrt{3}}{1\cdot 3} = \frac{1}{-\frac{\sqrt{3}}{2}}$$

$$\frac{2\sqrt{3}}{3} \neq -\frac{2}{\sqrt{3}} = -\frac{2\sqrt{3}}{3}$$

$$\tan(x) + \sqrt{3} = \sec(x)$$

$$-\frac{\sqrt{3}}{3} + \sqrt{3} = \frac{1}{\frac{\sqrt{3}}{2}}$$

$$\boxed{\frac{2\sqrt{3}}{3} = \frac{2\sqrt{3}}{3}}$$ ✓