Final Exam Review

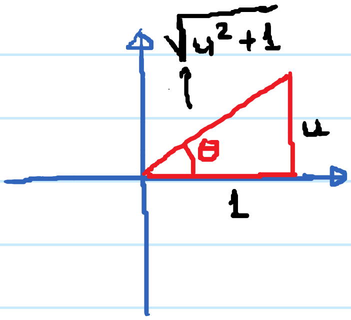
Monday, April 29, 2019 11:11 AM

MC

#1 $\cos\left(\boxed{\arctan u}\right)$ θ

Let $\theta = \arctan u \longrightarrow \boxed{\tan\theta} = u = \dfrac{u}{1}$

$\tan\theta = \dfrac{\text{opp.}}{\text{adj.}}$ u → opp, 1 → adj.



Want: $\cos\theta = \dfrac{\text{adj}}{\text{hyp}}$

$$\text{So, } \cos\theta = \frac{1}{\sqrt{u^2+1}} \cdot \frac{\sqrt{u^2+1}}{\sqrt{u^2+1}} = \boxed{\frac{\sqrt{u^2+1}}{u^2+1}}$$

$$\text{So, } \cos(\arctan u) = \frac{\sqrt{u^2+1}}{u^2+1}$$

#2 Given: $v = 40$ ft/s, $h = 5.7$ ft.

$$\theta = \sin^{-1}\left(\sqrt{\frac{v^2}{2v^2+64h}}\right)$$

$$= \sin^{-1}\left(\sqrt{\frac{(40)^2}{2(40)^2+64\cdot(5.7)}}\right)$$

$$\boxed{\theta \simeq 42^\circ}$$

(3) $2\sin^2 x + \sin x = 1$

$2\sin^2 x + \sin x - 1 = 0$

$(2\sin x - 1)(\sin x + 1) = 0$

$2\sin x - 1 = 0$ or $\sin x + 1 = 0$

$\sin x = \frac{1}{2}$ or $\sin x = -1$

$x = \frac{\pi}{6}, \frac{5\pi}{6}$ $\quad x = \frac{3\pi}{2}$

All solutions: $\boxed{\frac{\pi}{6} + n\cdot 2\pi, \frac{5\pi}{6} + n\cdot 2\pi, \frac{3\pi}{2} + n\cdot 2\pi}$

#4

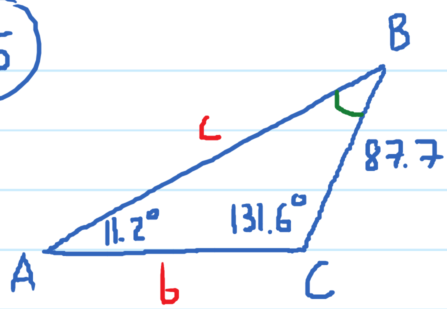
$\sin^2 x + \sin x = 0 \rightarrow \sin x(\sin x + 1) = 0$

$\sin x = 0$; $\sin x = -1$

$x = 0, \pi$ $\quad x = \frac{3\pi}{2}$

$x = \underbrace{n\cdot 2\pi, \pi + n\cdot 2\pi}_{\text{Simplify to: } \boxed{x = n\pi}}$; $\boxed{\frac{3\pi}{2} + n\cdot 2\pi}$

#5



$$\angle B = 180^\circ - (11.2^\circ + 131.6^\circ)$$

$$\boxed{\angle B = 37.2^\circ}$$

$$\frac{a}{\sin(A)} = \frac{b}{\sin(B)} \rightarrow b = \frac{a \cdot \sin(B)}{\sin(A)} = \frac{87.7 \sin(37.2^\circ)}{\sin(11.2^\circ)}$$

$$\boxed{b \approx 273}$$

$$\frac{a}{\sin(A)} = \frac{c}{\sin(C)} \rightarrow c = \frac{a \sin(C)}{\sin(A)} = \frac{87.7 \sin(131.6^\circ)}{\sin(11.2^\circ)}$$

$$\boxed{c = 337.6}$$

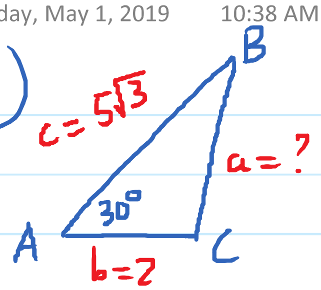
#6

$$\text{Area} = \frac{1}{2} bc \sin A$$

$$= \frac{1}{2}(10.3)(6.8)\sin(28.9^\circ)$$

$$= \boxed{16.9}$$

#7



$$a^2 = b^2 + c^2 - 2bc\cos(A)$$

$$a^2 = (2)^2 + (5\sqrt{3})^2 - 2 \cdot 2 \cdot 5\sqrt{3} \cdot \cos(30°)$$

$$a = \sqrt{(2)^2 + (5\sqrt{3})^2 - 20\sqrt{3}\cos(30°)} = \boxed{7}$$

#8

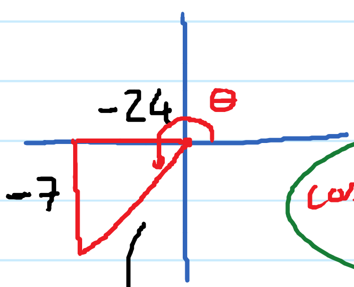
$$\sin(\overset{\theta}{\boxed{8x}})\cos(\overset{\theta}{\boxed{8x}})$$

$$= \boxed{\frac{1}{2}\sin(16x)}$$

$$\boxed{\begin{aligned} \sin(2\theta) &= 2\sin(\theta)\cos(\theta) \\ \frac{1}{2}\sin(2\theta) &= \sin(\theta)\cos(\theta) \end{aligned}}$$

#9

$$\cos(2\theta) = 2\cos^2\theta - 1 \longrightarrow \text{Find } \cos\theta.$$



$$\cos\theta = -\frac{24}{25}$$

$$\sqrt{(-7)^2 + (-24)^2} = 25$$

$$\text{So, } \cos(2\theta) = 2 \cdot \left(-\frac{24}{25}\right)^2 - 1 = \boxed{\frac{527}{625}}$$

Wednesday, May 1, 2019 10:45 AM

#10

sine of a difference

$$\sin\left(\frac{\pi}{4}-x\right) = \sin\frac{\pi}{4}\cos x - \cos\frac{\pi}{4}\sin x$$

$$= \boxed{\frac{\sqrt{2}}{2}\cos x - \frac{\sqrt{2}}{2}\sin x}$$

#11

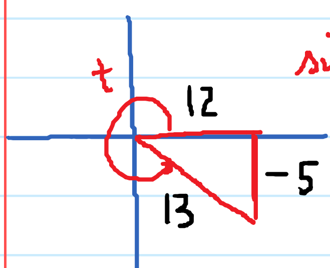
cosine of a difference

$$\cos(s-t) = \underbrace{\boxed{\cos(s)}\boxed{\cos(t)}}_{\text{given}} + \underbrace{\boxed{\sin(s)\sin(t)}}_{\text{missing}}$$

$\sin(s) = \frac{3}{5}$

$$= -\frac{4}{5}\cdot\frac{12}{13} + \frac{3}{5}\cdot\left(-\frac{5}{13}\right)$$

$$= \boxed{-\frac{63}{65}}$$



$\sin(t) = -\frac{5}{13}$

#12

$$\frac{\sin x\cos x}{\tan x} = \frac{\sin x\cos x}{\frac{\sin x}{\cos x}}$$

Quotient Identity

→ divide by fraction = multiply by reciprocal

$$= \cancel{\sin x}\cos x\cdot\frac{\cos x}{\cancel{\sin x}} = \boxed{\cos^2 x}$$

S. A.

#13 $\cos(-x)\cos x - \sin(-x)\sin x$

Even/odd

$= \cos x \cdot \cos x + \sin x \cdot \sin x$

$= \cos^2 x + \sin^2 x = \boxed{1}$.

#14 $\cos\left(2\underbrace{\boxed{\arcsin\left(\frac{1}{4}\right)}}_{y}\right) = \cos(2y)$

$= 1 - 2\sin^2 y$

$\boxed{y = \arcsin\left(\frac{1}{4}\right) \rightarrow \sin(y) = \frac{1}{4}}$

$= 1 - 2\cdot\left(\frac{1}{4}\right)^2$

$= 1 - 2\cdot\frac{1}{16} = \boxed{\frac{7}{8}}$.

#15

$$R = \frac{1}{32}\cdot v^2 \cdot \sin(2\theta)$$

Given: $R = 3000$; $V = 2000$. Want: θ

$$3000 = \frac{1}{32}\cdot(2000)^2 \cdot \sin(2\theta)$$

$$\sin(2\theta) = \frac{3000}{\frac{1}{32}\cdot(2000)^2} = \frac{3}{125}$$

$2\theta = \sin^{-1}\left(\frac{3}{125}\right)$. So, $\theta = \dfrac{\sin^{-1}\left(\frac{3}{125}\right)}{2} = \boxed{0.69^\circ}$

16 $\dfrac{\sin 22^\circ}{1+\cos 22^\circ}$ θ θ

$$\boxed{\tan\left(\frac{\theta}{2}\right) = \frac{\sin\theta}{1+\cos\theta}}$$

$= \boxed{\tan(11^\circ)}$

#17 $\cos(2\theta) = -\frac{3}{8}$ and $\frac{\pi}{2} < \theta < \pi$ QII

Find $\sin\theta$.

Double-angle identity for cosine

$\cos(2\theta) = 1 - 2\sin^2\theta$

$-\frac{3}{8} = 1 - 2\sin^2\theta$

$-\frac{3}{8} - 1 = -2\sin^2\theta$

$-\frac{11}{8} = -2\sin^2\theta$

$\sin^2\theta = \frac{11}{16} \rightarrow \sin\theta = \pm\frac{\sqrt{11}}{4}$

Since θ is in QII, $\boxed{\sin\theta = \frac{\sqrt{11}}{4}}$

18

$$\frac{\sin\theta}{1+\sin\theta} - \frac{\sin\theta}{1-\sin\theta}$$

$$\frac{\sin\theta}{1+\sin\theta}\cdot\frac{1-\sin\theta}{1-\sin\theta} - \frac{\sin\theta}{1-\sin\theta}\cdot\frac{1+\sin\theta}{1+\sin\theta}$$

$$= \frac{\sin\theta\cdot(1-\sin\theta) - \sin\theta(1+\sin\theta)}{1-\sin^2\theta}$$

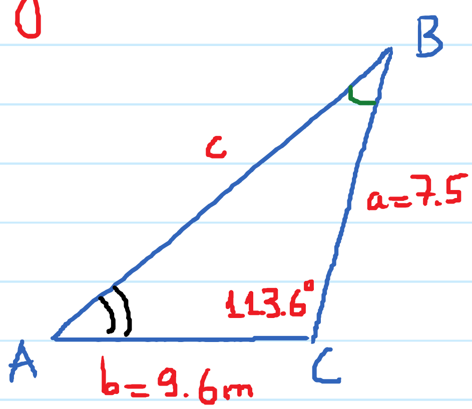
$$= \frac{\cancel{\sin\theta} - \sin^2\theta - \cancel{\sin\theta} - \sin^2\theta}{\cos^2\theta}$$

Pythagorean identity

$$= \frac{-2\sin^2\theta}{\cos^2\theta} = \boxed{-2\tan^2\theta}$$

Essay

#19



$$c^2 = a^2 + b^2 - 2ab\cos C$$

$$c^2 = (7.5)^2 + (9.6)^2 - 2\cdot(7.5)(9.6)\cdot\cos(113.6^\circ)$$

$$c = \boxed{14.35}$$

$$\cos(B) = \frac{a^2+c^2-b^2}{2ac} = \frac{(7.5)^2+(14.35)^2-(9.6)^2}{2\cdot(7.5)(14.35)} = \boxed{\cdots\cdots}$$

$$B = \cos^{-1}(\cdots\cdots) = \boxed{38^\circ}$$

$\angle A = 180^\circ - (113.6^\circ + 38^\circ) = \boxed{28.4^\circ}$

#20

$\sqrt{3}\sec(2\theta) = 2.$ $\quad 0^\circ \le \theta < 360^\circ$

$\rightarrow \boxed{0^\circ \le 2\theta < 720^\circ}$

$\sec(2\theta) = \dfrac{2}{\sqrt{3}}$

$\cos(2\theta) = \dfrac{\sqrt{3}}{2}$

$\rightarrow$ unit circle : $2\theta = 30^\circ, 330^\circ, 390^\circ, 690^\circ$

$\longrightarrow \boxed{\theta = 15^\circ, 165^\circ, 195^\circ, 345^\circ}$