Test 3 Review

Wednesday, April 3, 2019 10:27 AM

MC

1. $\cot(\theta) = -\sqrt{15}$; θ is in QII

   Find $\csc(\theta)$.

Identity: $\cot^2(\theta) + 1 = \csc^2(\theta)$

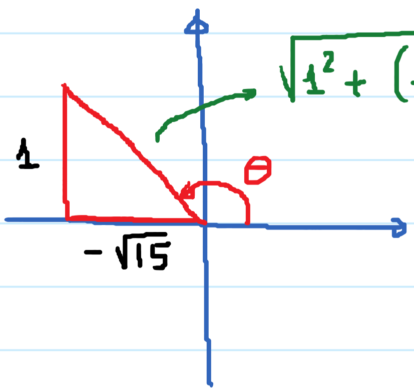
$$(-\sqrt{15})^2 + 1 = \csc^2(\theta)$$

So, $\csc^2(\theta) = 16 \longrightarrow \csc(\theta) = \pm 4$

θ is in QII, so $\sin(\theta) > 0$, so $\csc(\theta) = \dfrac{1}{\sin(\theta)} > 0$

Ans: $\csc(\theta) = 4$.

2nd method



$\sqrt{1^2 + (-\sqrt{15})^2} = \sqrt{16} = 4$

$$\csc(\theta) = \frac{1}{\sin(\theta)} = \frac{1}{\frac{1}{4}} = 4$$

2. $\sec^2(x) - 1 = \tan^2(x)$

3 $$\frac{\sin x}{\tan x} = \frac{\sin x}{\frac{\sin x}{\cos x}} = \frac{\cancel{\sin x}}{1} \cdot \frac{\cos x}{\cancel{\sin x}} = \boxed{\cos x}$$

Quotient Identity

4 $$\frac{\sin\theta}{\cos\theta} \cdot \frac{\sin\theta}{\sin\theta} + \frac{\cos\theta}{\sin\theta} \cdot \frac{\cos\theta}{\cos\theta}$$

$$= \frac{\sin^2\theta}{\sin\theta \cdot \cos\theta} + \frac{\cos^2\theta}{\sin\theta\cos\theta} = \frac{\boxed{\sin^2\theta + \cos^2\theta}}{\sin\theta\cos\theta}$$

$$= \frac{1}{\sin\theta\cos\theta} = \frac{1}{\sin\theta} \cdot \frac{1}{\cos\theta} = \boxed{\csc\theta\sec\theta}$$

5 $$\cos\left(\theta + \frac{\pi}{2}\right) = \cos\theta \underbrace{\boxed{\cos\frac{\pi}{2}}}_{0} - \sin\theta \underbrace{\boxed{\sin\frac{\pi}{2}}}_{1}$$

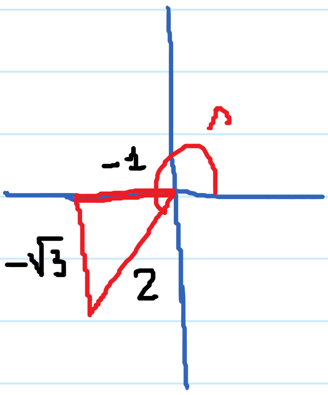
Cosine of a sum

$$= -\sin\theta$$

6 $$\cos(s-t) = \underbrace{\boxed{\cos(s)}}_{\text{given}}\underbrace{\boxed{\cos(t)}}_{\text{given}} + \underbrace{\boxed{\sin(s)}\boxed{\sin(t)}}_{\text{missing}}$$

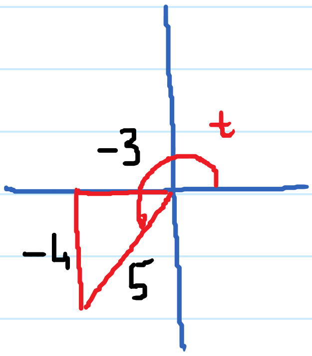
cosine of a difference

* Find $\sin(s)$ given $\cos(s) = -\frac{1}{2}$; s is in Q III



$$\sin(s) = -\frac{\sqrt{3}}{2}$$

* Find $\sin(t)$ given $\cos(t) = -\frac{3}{5}$; t is in Q III



$$\sin(t) = -\frac{4}{5}$$

So, $\cos(s-t) = \left(-\frac{1}{2}\right)\left(-\frac{3}{5}\right) + \left(-\frac{\sqrt{3}}{2}\right)\left(-\frac{4}{5}\right)$

$$= \frac{3}{10} + \frac{4\sqrt{3}}{10} = \boxed{\frac{3+4\sqrt{3}}{10}}$$

(7) Done on the board.

(8) $\sin\left(\frac{\pi}{4} - x\right) = \sin\left(\frac{\pi}{4}\right)\cos(x) - \sin(x)\cos\left(\frac{\pi}{4}\right)$

Sine of a difference

$$= \boxed{\frac{\sqrt{2}}{2}\cos(x) - \frac{\sqrt{2}}{2}\sin(x)}$$

9 $\sin(\theta) = \dfrac{12}{13}$, $\cos(\theta) > 0$

Find $\cos(2\theta)$

By double angle identity for cosine

$$\cos(2\theta) = 1 - 2\sin^2(\theta)$$

$$= 1 - 2\cdot\left(\frac{12}{13}\right)^2 = \boxed{-\frac{119}{169}}$$

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10 $4\sin(\boxed{2x})\cos(\boxed{2x}) = 2\cdot\boxed{2\sin(2x)\cos(2x)}$

$$= \boxed{2\sin(4x)}$$

Double angle identity for sine

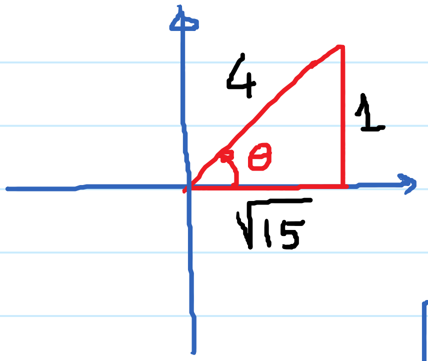
11 Find $\cos\left(\dfrac{\theta}{2}\right)$; given $\sin(\theta) = \dfrac{1}{4}$ and $\underbrace{0 < \theta < 90^\circ}$

$\underbrace{0 < \dfrac{\theta}{2} < 45^\circ}$

$\dfrac{\theta}{2}$ is in QI

$$\cos\left(\frac{\theta}{2}\right) = \pm\sqrt{\frac{1+\cos(\theta)}{2}}$$

$$\longrightarrow \cos\left(\frac{\theta}{2}\right) = \sqrt{\frac{1+\cos(\theta)}{2}}$$



$$\cos(\theta) = \frac{\sqrt{15}}{4}$$

$$\text{So, } \cos\left(\frac{\theta}{2}\right) = \sqrt{\frac{\frac{4\cdot 1}{4\cdot 1} + \frac{\sqrt{15}}{4}}{2}} = \sqrt{\frac{\frac{4+\sqrt{15}}{4}}{2}}$$

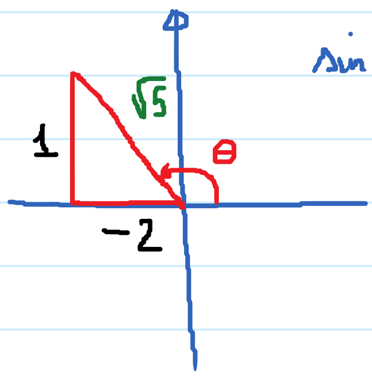
$$= \sqrt{\frac{4+\sqrt{15}}{8}} = \frac{\sqrt{4+\sqrt{15}}}{2\sqrt{2}} \cdot \frac{\sqrt{2}}{\sqrt{2}}$$

$$= \boxed{\frac{\sqrt{8+2\sqrt{15}}}{4}}$$

12 Half angle identity for tangent

$$\frac{\sin(52^\circ)}{1+\cos(52^\circ)} = \tan(26^\circ)$$

13



$$\sin(\theta) = \frac{1}{\sqrt{5}} = \frac{\sqrt{5}}{5}$$

14 Cosine of a difference:

$$\cos\frac{7\pi}{12}\cos\frac{5\pi}{12}+\sin\frac{7\pi}{12}\sin\frac{5\pi}{12}$$

$$=\cos\left(\frac{7\pi}{12}-\frac{5\pi}{12}\right)=\cos\left(\frac{2\pi}{12}\right)=\cos\left(\frac{\pi}{6}\right)=\boxed{\frac{\sqrt{3}}{2}}$$

15 $$\tan\left(x-\frac{\pi}{3}\right)=\frac{\tan(x)-\tan\left(\frac{\pi}{3}\right)}{1+\tan(x)\tan\left(\frac{\pi}{3}\right)}$$

$$=\boxed{\frac{\tan(x)-\sqrt{3}}{1+\sqrt{3}\tan x}}$$

16 $$\sin(s-t)=\underbrace{\boxed{\sin(s)}}_{-\frac{1}{2}}\cdot\cos(t)-\cos(s)\cdot\underbrace{\boxed{\sin(t)}}_{\frac{1}{4}}$$

Using triangle, $\cos(s)=\frac{\sqrt{3}}{2}$; $\cos(t)=-\frac{\sqrt{15}}{4}$

$$=-\frac{1}{2}\cdot\left(-\frac{\sqrt{15}}{4}\right)-\frac{\sqrt{3}}{2}\cdot\frac{1}{4}$$

$$=\frac{\sqrt{15}}{8}-\frac{\sqrt{3}}{8}=\boxed{\frac{\sqrt{15}-\sqrt{3}}{8}}$$

(17) $\cos^2(4x) - \sin^2(4x) = \cos(8x)$

Double angle identity for cosine

(18) Half angle identity:

$\underbrace{\cos\left(\frac{A}{2}\right)} = \sqrt{\frac{1+\cos(A)}{2}}$ $\rightarrow 26^\circ$

$\cos(13^\circ)$

(19) $\frac{1-\sec(\theta)}{\tan(\theta)} + \frac{\tan(\theta)}{1-\sec(\theta)} = -2\csc(\theta)$

LHS $= \frac{1-\sec(\theta)}{\tan(\theta)} \cdot \frac{1-\sec(\theta)}{1-\sec(\theta)} + \frac{\tan(\theta)}{1-\sec(\theta)} \cdot \frac{\tan(\theta)}{\tan(\theta)}$

$= \frac{(1-\sec(\theta))^2 + \tan^2(\theta)}{\tan(\theta)\cdot(1-\sec(\theta))}$

$= \frac{\boxed{1} - 2\sec(\theta) + \sec^2(\theta) \boxed{+\tan^2(\theta)}}{\tan(\theta)(1-\sec(\theta))}$

$= \frac{-2\sec(\theta) + \sec^2(\theta) + \sec^2(\theta)}{\tan(\theta)(1-\sec(\theta))}$

$$= \frac{-2\sec\theta + 2\sec^2\theta}{\tan(\theta)\,(1-\sec(\theta))}$$

$$= \frac{-2\sec\theta\,(\cancel{1-\sec\theta})}{\tan\theta\,(\cancel{1-\sec\theta})}$$

$$= -\frac{2\sec\theta}{\tan\theta} = -\frac{\frac{2}{\cos\theta}}{\frac{\sin\theta}{\cos\theta}}$$

$$= -\frac{2}{\cancel{\cos\theta}}\cdot\frac{\cancel{\cos\theta}}{\sin\theta} = -\frac{2}{\sin\theta} = -2\csc(\theta) = RHS.$$

(20) $\cos(4\theta) = \cos^4(\theta) - 6\sin^2(\theta)\cos^2(\theta) + \sin^4(\theta)$

$$LHS = \cos(2\cdot 2\theta) = \cos^2(2\theta) - \sin^2(2\theta)$$

$$= [\cos(2\theta)]^2 - [\sin(2\theta)]^2$$

$$= [\cos^2(\theta) - \sin^2(\theta)]^2 - [2\sin(\theta)\cos(\theta)]^2$$

$$= (\cos^2(\theta) - \sin^2(\theta))\cdot(\cos^2(\theta) - \sin^2(\theta))$$
$$- 4\sin^2(\theta)\cos^2(\theta)$$

$$= \cos^4(\theta) - 2\sin^2(\theta)\cos^2(\theta) + \sin^4(\theta) - 4\sin^2(\theta)\cos^2(\theta)$$

$$= \cos^4(\theta) - 6\sin^2(\theta)\cos^2(\theta) + \sin^4(\theta) = RHS.$$