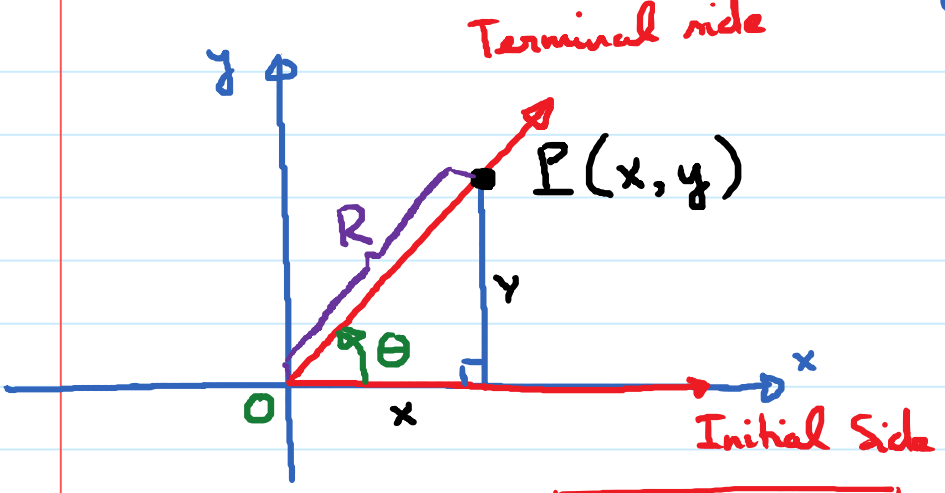
1.3. Trigonometric Functions

Tuesday, January 22, 2019 8:21 AM



(theta)
θ: angle in standard position.
P: arbitrary point on the terminal side.
R: distance from O to P.

$$R^2 = x^2 + y^2 \longrightarrow \boxed{R = \sqrt{x^2 + y^2}}$$

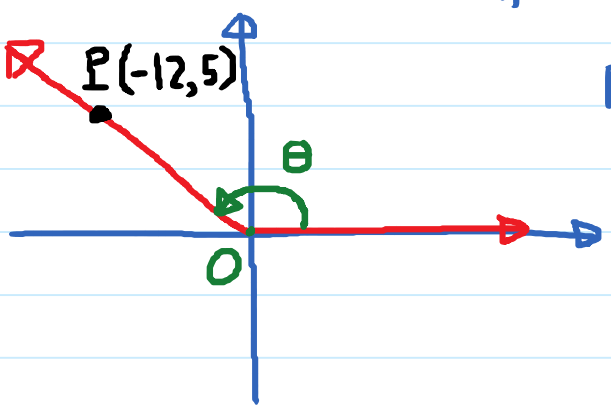
Def. of the six trig functions of the angle θ:

$\sin\theta := \frac{y}{R}$ (sine); $\cos\theta := \frac{x}{R}$ (cosine); $\tan\theta := \frac{y}{x}$ $(x \neq 0)$ (tangent)

$\cot\theta := \frac{x}{y}$ (cotangent) $(y \neq 0)$; $\sec\theta = \frac{R}{x}$ (secant) $(x \neq 0)$; $\csc\theta = \frac{R}{y}$ (cosecant) $(y \neq 0)$

E.g. The terminal side of an angle θ in standard position passes through the point (−12, 5).

Find the values of the six trig functions of θ.



Find R. $R = \sqrt{x^2 + y^2}$

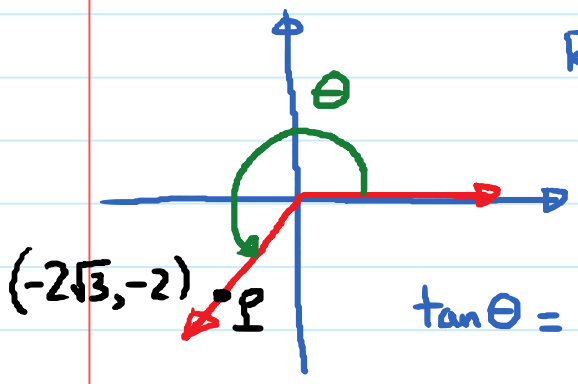
$R = \sqrt{(-12)^2 + (5)^2} = 13.$

$\sin\theta = \frac{y}{R} = \boxed{\frac{5}{13}}$; $\cos\theta = \frac{x}{R} = \frac{-12}{13} = \boxed{-\frac{12}{13}}$

$\tan\theta = \frac{y}{x} = \frac{5}{-12} = \boxed{-\frac{5}{12}}$; $\sec\theta = \frac{R}{x} = \frac{13}{-12} = \boxed{-\frac{13}{12}}$

$\csc\theta = \frac{R}{y} = \boxed{\frac{13}{5}}$; $\cot\theta = \frac{x}{y} = \frac{-12}{5} = \boxed{-\frac{12}{5}}$

E.g. Same question, the point now is $(-2\sqrt{3}, -2)$



$R = \sqrt{(-2\sqrt{3})^2 + (-2)^2} = 4.$

$\sin\theta = -\frac{1}{2}$; $\cos\theta = -\frac{2\sqrt{3}}{4} = \boxed{-\frac{\sqrt{3}}{2}}$

$\tan\theta = \frac{-2}{-2\sqrt{3}} = \frac{1}{\sqrt{3}} \cdot \frac{\sqrt{3}}{\sqrt{3}} = \boxed{\frac{\sqrt{3}}{3}}$

$\cot\theta = \frac{-2\sqrt{3}}{-2} = \boxed{\sqrt{3}}$

$\sec\theta = -\frac{2}{\sqrt{3}} = \boxed{-\frac{2\sqrt{3}}{3}}$; $\csc\theta = \boxed{-2}$

Values of the 6 trig Functions of quadrantal angles.

θ	Pt on term.	$\sin\theta$	$\cos\theta$	$\tan\theta$	$\cot\theta$	$\sec\theta$	$\csc\theta$
$0°$	$(1,0)$	0	1	0	undef.	1	undef
$90°$	$(0,1)$	1	0	undef	0	undef	1
$180°$	$(-2,0)$ $R=2$	0	-1	0	undef	-1	undef
$270°$	$(0,-1)$	-1	0	undef	0	undef	-1

Note: Coterminal angles have the same trig function values

$90° \longrightarrow 450°$ $\quad \cos 450° = \cos 90° = 0$, $\csc 450° = \csc 90° = 1$

$90° \longrightarrow -270°$ $\quad \sin(-270°) = \sin 90° = 1$

table

You know the values of trig functions of all angles of the form $90° + n \cdot 360°$ where

$n = \ldots -2, -1, 0, 1, 2$

Evaluate expressions:

$$\cos^2(360^\circ) - \sin^2(360^\circ)$$

$$= [\cos(360^\circ)]^2 - [\sin(360^\circ)]^2$$

360° is coterminal with 0°.

$\cos 0^\circ = 1$

$\sin 0^\circ = 0$

$$= [1]^2 - [0]^2 = \boxed{1}.$$