1.4. Using the Definitions of Trig Functions

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Reciprocal Identities.

$$\sin\theta = \frac{1}{\csc\theta} \; ; \quad \cos\theta = \frac{1}{\sec\theta} \; ; \quad \tan\theta = \frac{1}{\cot\theta}$$

$$\csc\theta = \frac{1}{\sin\theta} \; ; \quad \sec\theta = \frac{1}{\cos\theta} \; ; \quad \cot\theta = \frac{1}{\tan\theta}$$

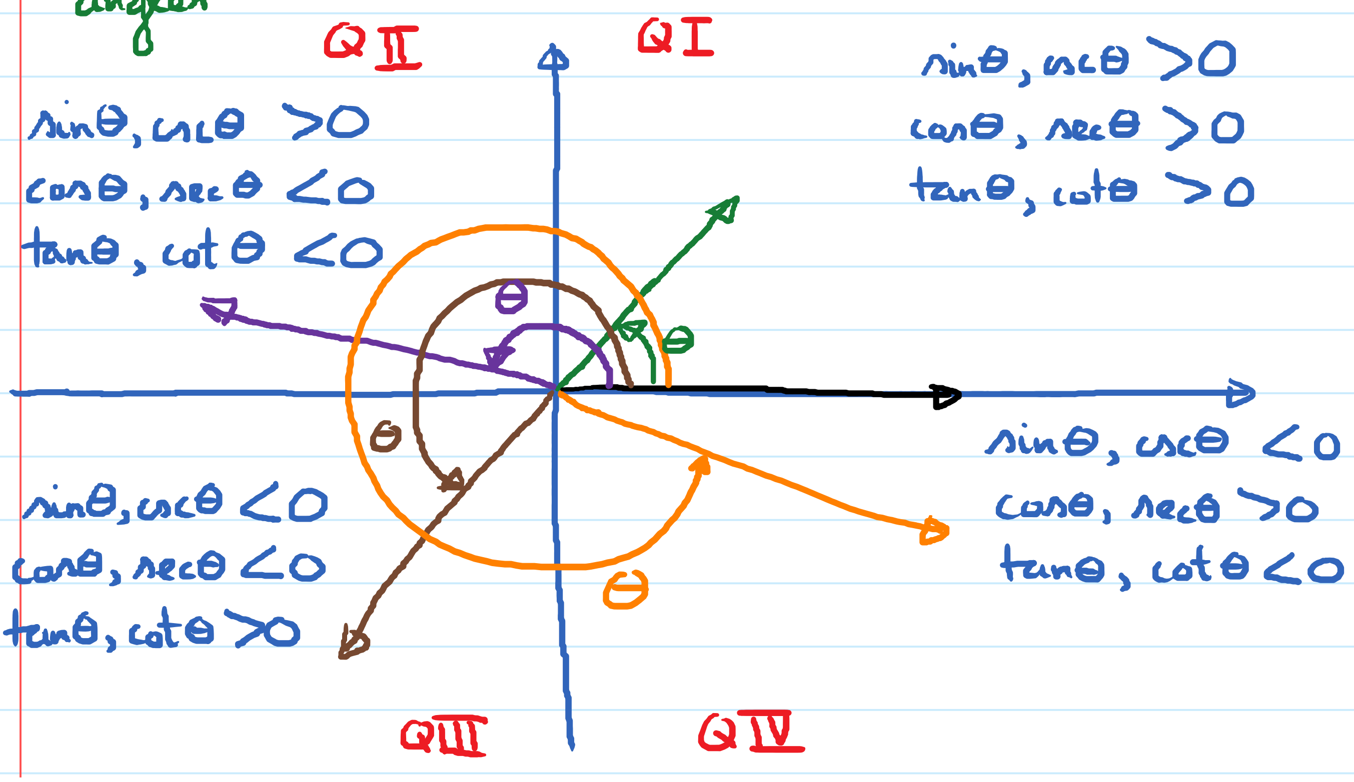
E.g. Given that $\cot\theta = -4$. Find $\tan\theta$.

$$\tan\theta = -\frac{1}{4}$$

E.g. $\cos\theta = -\frac{2}{\sqrt{20}}$. Find $\sec\theta$.

$$\sec\theta = -\frac{\sqrt{20}}{2} = -\frac{\cancel{2}\sqrt{5}}{\cancel{2}} = -\sqrt{5}$$

Determine the signs of trig functions of non quadrantal angles



E.g. $\cos\theta < 0$, $\sin\theta < 0$

$\rightarrow$ θ is in QIII.

E.g. $\cos\theta > 0$, $\sec\theta > 0$

$\rightarrow$ θ is in QI or QIV.

E.g. $\cot\theta < 0$, $\sec\theta < 0$.

$\rightarrow$ θ is in QII.

E.g. $-115°$

$\sin(-115°) < 0$; $\cos(-115°) < 0$,

$\tan(-115°) > 0$

E.g. $855° \rightarrow$ QII

$\sin(855°) > 0$, $\cos(855°) < 0$, $\tan(855°) < 0$

Pythagorean Identities.

$$\sin^2\theta + \cos^2\theta = 1$$

$$\sin^2\theta = 1 - \cos^2\theta$$

$$\cos^2\theta = 1 - \sin^2\theta$$

$$\tan^2\theta + 1 = \sec^2\theta \rightarrow \sec^2\theta - \tan^2\theta = 1$$

$$\tan^2\theta + 1 = \sec^2\theta \rightarrow \tan^2\theta = \sec^2\theta - 1$$

$$\cot^2\theta + 1 = \csc^2\theta \rightarrow \csc^2\theta - \cot^2\theta = 1$$

$$\cot^2\theta + 1 = \csc^2\theta \rightarrow \cot^2\theta = \csc^2\theta - 1$$

Quotient Identities:

$$\boxed{\tan\theta = \frac{\sin\theta}{\cos\theta} \quad ; \quad \cot\theta = \frac{\cos\theta}{\sin\theta}}$$

$$\tan\theta = \frac{y/R}{x/R} = \frac{\sin\theta}{\cos\theta}$$

E.g. Given: $\cos\theta = \frac{4}{5}$ and θ is in Q IV

Find $\sin\theta$.

$$\sin^2\theta = 1 - \cos^2\theta = 1 - \left(\frac{4}{5}\right)^2 = \frac{1 \cdot 25}{1 \cdot 25} - \frac{16}{25}$$

$$= \frac{25 - 16}{25} = \frac{9}{25}$$

$\sin^2\theta = \frac{9}{25} \rightarrow \sin\theta = \pm\frac{3}{5}$

Since θ is in Q IV, $\sin\theta < 0$.

Hence, $\sin\theta = -\frac{3}{5}$.

E.g. Given $\sin\theta = \frac{1}{2}$ and θ is in Q II.

Find $\tan\theta$.

$\csc\theta = \frac{1}{\sin\theta} = 2$

$\cot^2\theta = \csc^2\theta - 1 = 4 - 1 = 3$

$\cot\theta = \pm\sqrt{3}$. θ is in Q II, so $\cot\theta = -\sqrt{3}$

$\tan\theta = \frac{1}{\cot\theta} = \frac{1}{-\sqrt{3}} = -\frac{1}{\sqrt{3}} \cdot \frac{\sqrt{3}}{\sqrt{3}} = \boxed{-\frac{\sqrt{3}}{3}}$

E.g. $\cos\theta = -\frac{\sqrt{3}}{2}$, θ is in Q III

Find the values of the rest of the trig functions of θ.

$\sin^2\theta = 1 - \cos^2\theta = 1 - \left(-\frac{\sqrt{3}}{2}\right)^2 = 1 - \frac{3}{4}$

$\sin^2\theta = \frac{1}{4} \rightarrow \sin\theta = \pm\frac{1}{2}$

θ is in Q III, So, $\boxed{\sin\theta = -\frac{1}{2}}$

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$$\tan\theta = \frac{\sin\theta}{\cos\theta} = \frac{\cancel{-}\frac{1}{2}}{\cancel{-}\frac{\sqrt{3}}{2}} = \frac{1}{2}\cdot\frac{2}{\sqrt{3}} = \frac{\cancel{2}}{\cancel{2}\sqrt{3}}$$

$$\tan\theta = \frac{1}{\sqrt{3}}\cdot\frac{\sqrt{3}}{\sqrt{3}} = \frac{\sqrt{3}}{3}.$$

$$\boxed{\tan\theta = \frac{\sqrt{3}}{3}} \longrightarrow \boxed{\cot\theta = \frac{3}{\sqrt{3}} = \sqrt{3}}$$

$$\boxed{\csc\theta = -2} \qquad \boxed{\sec\theta = -\frac{2}{\sqrt{3}}\cdot\frac{\sqrt{3}}{\sqrt{3}} = -\frac{2\sqrt{3}}{3}}$$

The Range Values of Trig Functions.

Function	Range	
$\sin\theta, \cos\theta$	$[-1, 1]$	$-1 \le \sin\theta, \cos\theta \le 1$
$\csc\theta, \sec\theta$	$(-\infty, -1] \cup [1, \infty)$	
$\tan\theta, \cot\theta$	$(-\infty, \infty)$	