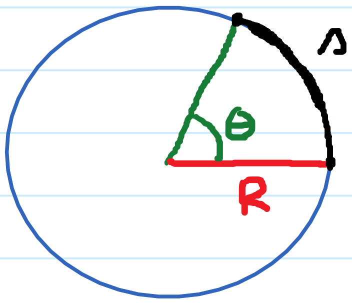
3.2. Applications of Radian Measures

Thursday, February 14, 2019 9:10 AM

Arc length

$\boxed{s = R \cdot \theta}$



Note: θ must be measured in radians

E.g. Find the length of the arc intercepted by a central angle $\theta = 210^\circ$. Given $R = 25.6$ (cm)

Convert θ to radians : $\theta = 210 \cdot \dfrac{\pi}{180} = \dfrac{7\pi}{6}$ (radians)

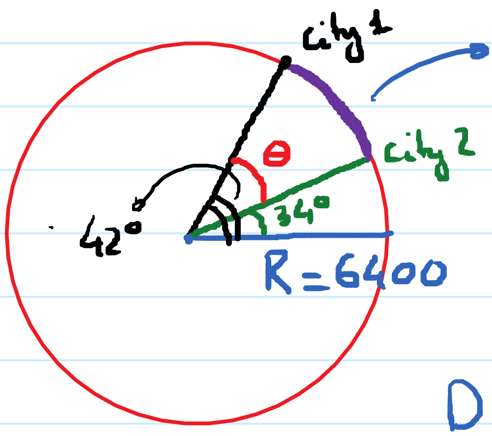
$s = R \cdot \theta = (25.6) \cdot \left(\dfrac{7\pi}{6}\right) \simeq 93.83$ (cm)

E.g. city 1 : latitude $= 42^\circ$ N

city 2 : latitude $= 34^\circ$ N

Radius of earth $R = 6400$ km

Q: Find distance between the 2 cities



$s = ?$

$\theta = 42^\circ - 34^\circ = 8^\circ$

$\theta = 8^\circ \cdot \dfrac{\pi}{180^\circ} = \dfrac{2\pi}{45}$.

Distance $= s = (6400)\cdot\left(\dfrac{2\pi}{45}\right) = \ldots$

E.g. (2 gears)

Step 1: Distance traveled by the smaller gear.

$\theta = 225^\circ$; $R = 2.5$ (cm) ; $s = ?$

$s = R\cdot\theta = (2.5)\cdot(225)\cdot\dfrac{\pi}{180} = \dfrac{25}{8}\pi$

Step 2: Angle that larger gear rotates.

$s = \dfrac{25}{8}\pi$, $R = 4.8$ (cm) ; θ

$\theta = \dfrac{s}{R} = \dfrac{\frac{25}{8}\pi}{4.8} = \dfrac{125}{192}\pi$

Convert to degrees:

$\dfrac{125\cancel{\pi}}{192}\cdot\dfrac{180}{\cancel{\pi}} \simeq \boxed{117.19^\circ}$

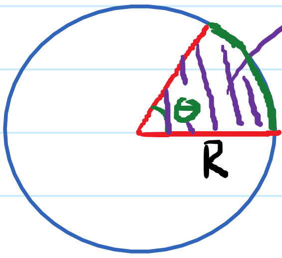
E.g.

$\theta = 51.6^\circ$; $s = 11.4$

$R = \dfrac{s}{\theta} = \dfrac{11.4}{(51.6)\cdot\left(\frac{\pi}{180}\right)} \simeq \boxed{12.66}$ cm.

Tuesday, February 19, 2019 8:37 AM

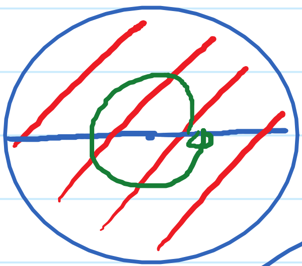
Area of a sector.



Area = ?

$$\boxed{A = \frac{1}{2}R^2\cdot\theta}$$

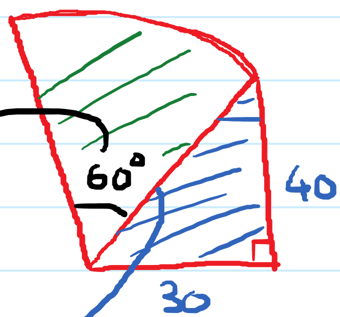
(Note: θ is measured in radian)



Area of whole circle		Central angle
πR^2	$\longleftrightarrow$	2π
? (Area of sector)	$\longleftrightarrow$	θ
$\frac{\cancel{\pi} R^2}{2\cancel{\pi}} = \frac{R^2}{2}$		1 radian
$\frac{R^2}{2}\cdot\theta$		θ radians

E.g.

$\frac{\pi}{3}$



50 (Pythagorean Theorem)

$A = ?$

Area of △ : $= \frac{1}{2}\cdot 30\cdot 40 = 600$

Area of ⌓ $= \frac{1}{2}\cdot(50)^2\cdot\frac{\pi}{3}$

Add them up