3.3. Unit Circle and Circular Functions
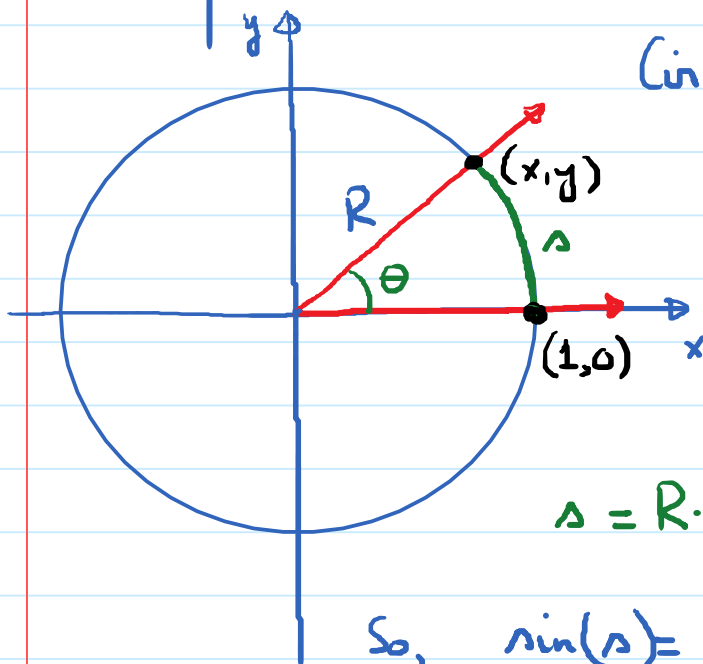
Recall
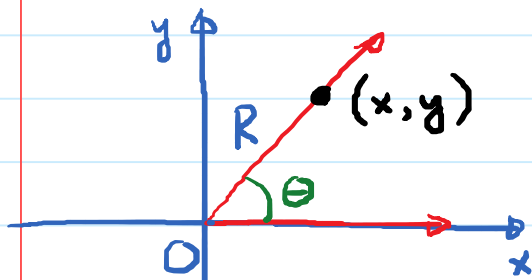
$$\sin\theta = \frac{y}{R} \quad ; \quad \cos\theta = \frac{x}{R}$$
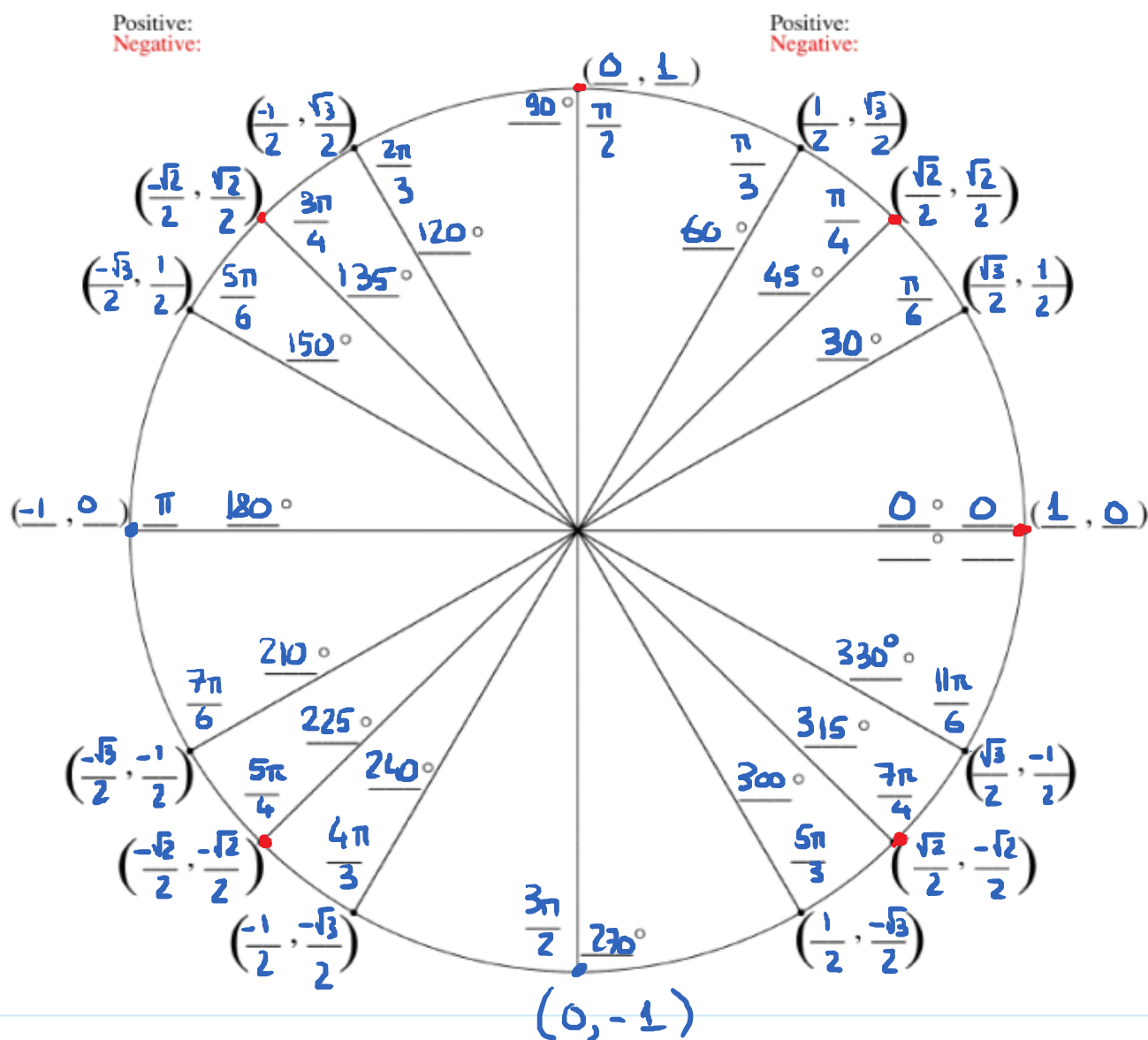
$$\tan\theta = \frac{y}{x}$$

Circle: center at $(0,0)$, radius $= 1$
(Unit circle)

$$\sin\theta = y \quad ; \quad \cos\theta = x$$

$$\tan\theta = \frac{y}{x}$$

$$s = R\cdot\theta = \theta$$

So, $\sin(s) = y$; $\cos(s) = x$; $\tan(s) = \frac{y}{x}$

Positive:
Negative:

Positive:
Negative:

$(0, 1)$

$90°$ $\frac{\pi}{2}$

$\left(\frac{-1}{2}, \frac{\sqrt{3}}{2}\right)$ $\frac{2\pi}{3}$

$\left(\frac{1}{2}, \frac{\sqrt{3}}{2}\right)$ $\frac{\pi}{3}$

$\left(\frac{-\sqrt{2}}{2}, \frac{\sqrt{2}}{2}\right)$ $\frac{3\pi}{4}$ $120°$

$60°$ $\frac{\pi}{4}$ $\left(\frac{\sqrt{2}}{2}, \frac{\sqrt{2}}{2}\right)$

$\left(\frac{-\sqrt{3}}{2}, \frac{1}{2}\right)$ $\frac{5\pi}{6}$ $135°$

$45°$ $\frac{\pi}{6}$ $\left(\frac{\sqrt{3}}{2}, \frac{1}{2}\right)$

$150°$

$30°$

$(-1, 0)$ π $180°$

$0°$ 0 $(1, 0)$

$210°$

$330°$

$\frac{7\pi}{6}$

$\frac{11\pi}{6}$

$225°$

$315°$

$\left(\frac{-\sqrt{3}}{2}, \frac{-1}{2}\right)$ $\frac{5\pi}{4}$ $240°$

$300°$ $\frac{7\pi}{4}$ $\left(\frac{\sqrt{3}}{2}, \frac{-1}{2}\right)$

$\left(\frac{-\sqrt{2}}{2}, \frac{-\sqrt{2}}{2}\right)$ $\frac{4\pi}{3}$

$\frac{5\pi}{3}$ $\left(\frac{\sqrt{2}}{2}, \frac{-\sqrt{2}}{2}\right)$

$\left(\frac{-1}{2}, \frac{-\sqrt{3}}{2}\right)$ $\frac{3\pi}{2}$ $270°$

$\left(\frac{1}{2}, \frac{-\sqrt{3}}{2}\right)$

$(0, -1)$

1st use of the unit circle :

We can find sine and cosine any angles related

one of the families :

(I) $\frac{\pi}{6}$; $\frac{5\pi}{6}$; $\frac{7\pi}{6}$; $\frac{11\pi}{6}$ (II) $\frac{\pi}{4}$, $\frac{3\pi}{4}$, $\frac{5\pi}{4}$, $\frac{7\pi}{4}$

(III) $\frac{\pi}{3}$, $\frac{2\pi}{3}$, $\frac{4\pi}{3}$, $\frac{5\pi}{3}$ (IV) 0, $\frac{\pi}{2}$, π, $\frac{3\pi}{2}$

by identifying which point on the unit circle that angle corresponds to and use the x and y coordinates of the points.

E.g. $\cos\left(\frac{37\pi}{6}\right)$

Coterminal: $\frac{37\pi}{6} - \frac{18 \cdot 2\pi}{6} = \frac{\pi}{6}$

So, $\cos\left(\frac{37\pi}{6}\right) = \frac{\sqrt{3}}{2}$.

E.g. $\sin\left(-\frac{7\pi}{6}\right)$

$-\frac{7\pi}{6} + \frac{2\pi \cdot 6}{1 \cdot 6} = -\frac{7\pi}{6} + \frac{12\pi}{6} = \boxed{\frac{5\pi}{6}}$

$\sin\left(-\frac{7\pi}{6}\right) = \frac{1}{2}$

2nd use of unit circle.

Find s given $\cos s$ or $\sin s$ or the value of any trig functions of s.

E.g. Find all the values of s in $[0, 2\pi)$ such that $\cos(s) = -\frac{1}{2}$.

Ans: $s = \dfrac{2\pi}{3}$ or $s = \dfrac{4\pi}{3}$

E.g. Find all the values of s in $\left[\dfrac{3\pi}{2}, 2\pi\right)$

such that $\sin s = -\dfrac{\sqrt{2}}{2}$

   Ans: $s = \dfrac{7\pi}{4}$.

E.g. Find all the values of s in $[0, 2\pi)$ such

that $\tan s = \dfrac{\sqrt{3}}{3}$

   $s = \dfrac{\pi}{6}$, $s = \dfrac{7\pi}{6}$.

E.g. Find all the values of s in $[0, 2\pi)$ such

that $\tan s = -1$

   $s = \dfrac{3\pi}{4}$ or $s = \dfrac{7\pi}{4}$.

Using calculator :

① Find trig function values:

   E.g. $\sin (0.6109)$ (No unit → rad)

      (Calculator is in rad mode)

   $\approx 0.5736\ldots$

② Given trig function values, find s.

E.g. Find s in $[0, \frac{\pi}{2})$ such that

$$\tan(s) = 0.2126.$$
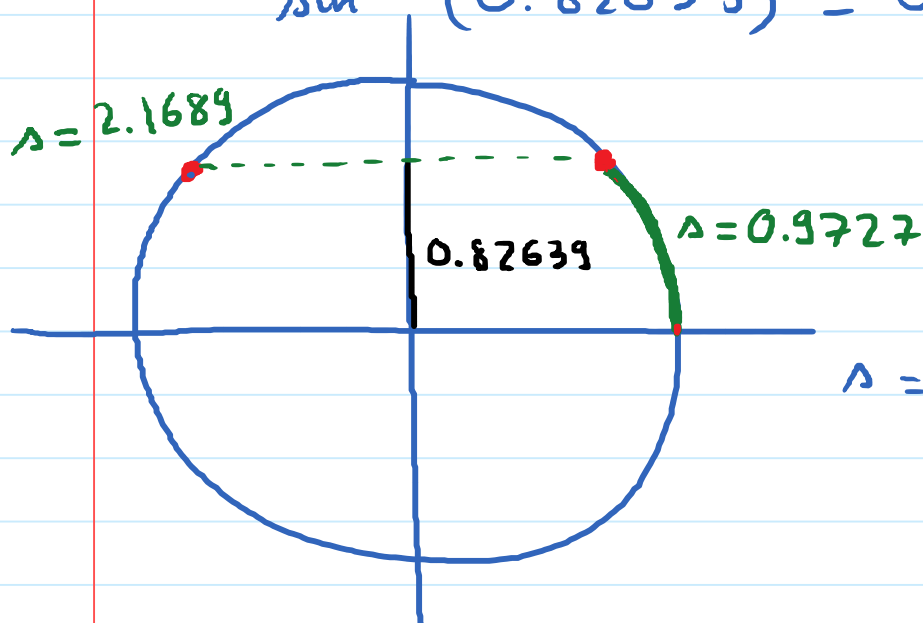
$2^{nd} \rightarrow \tan \rightarrow \tan^{-1}(0.2126)$

$\underbrace{}_{\tan^{-1}}$

$\rightarrow s = 0.20948 \ (rad)$

E.g. Find s in $[0, 2\pi)$ such that

$$\sin(s) = 0.82639.$$

$$\sin^{-1}(0.82639) \approx 0.9727 < \frac{\pi}{2}$$
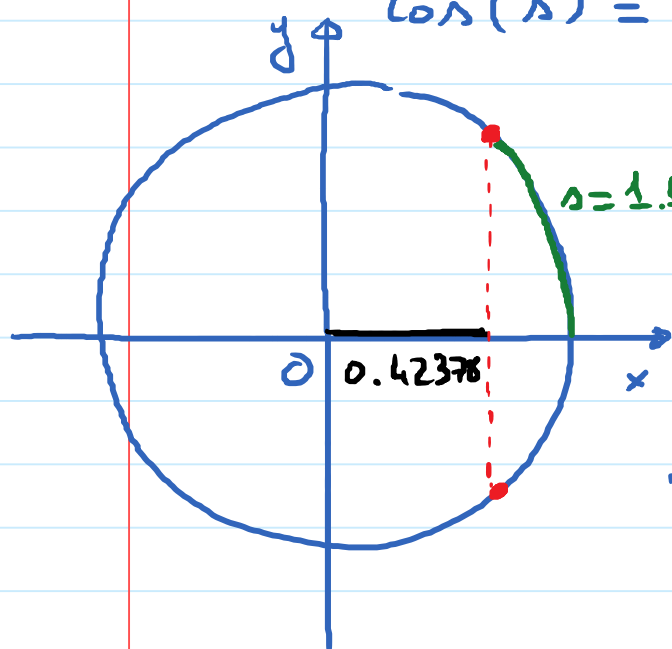
s = 2.1689

0.82639

s = 0.9727

1st solution:

s = 0.9727

2nd solution:

s = π − 0.9727 = 2.1689

E.g. Find s in $[0, 2\pi)$ such that

$$\cos(s) = 0.42378$$

1st solution:

$s = 1.1332$

$$s = \cos^{-1}(0.42378)$$

$$= 1.1332$$

2nd solution:

$$s = 2\pi - 1.1332$$

$$= 5.15.$$