4.1. Graphs of Sine and Cosine Functions

Tuesday, February 26, 2019 8:01 AM

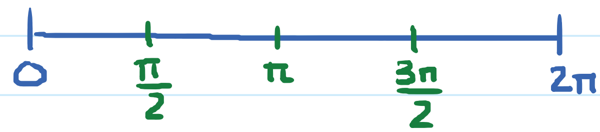
(1) Basic Sine and Cosine Curves.

$\boxed{y} = \sin \boxed{x}$

value of trig function ← y

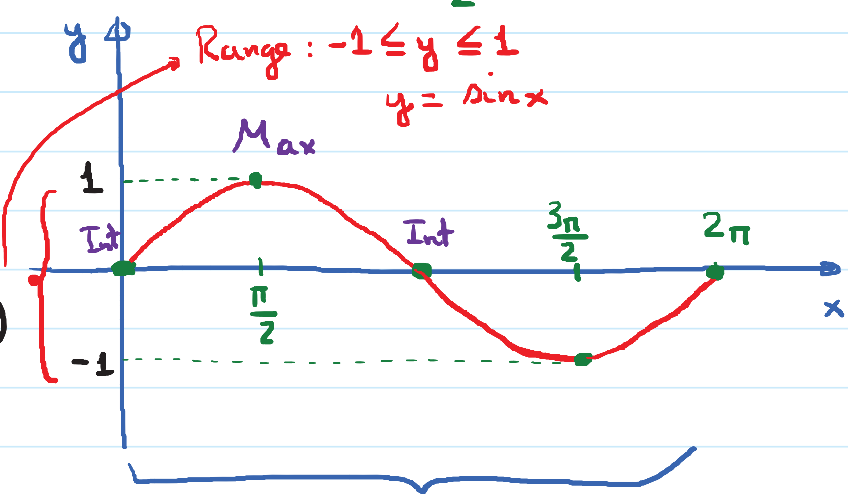
x → angle (measured in radian

To sketch the graph, we find 5 Key points where the angle x is in $[0, 2\pi]$. (Note: The graph repeats itself every 2π)

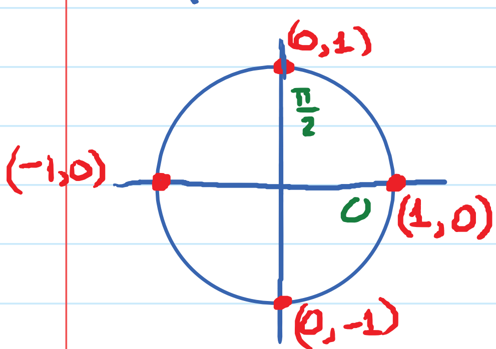


	x	$y = \sin x$	
Start ←	0	0	→ $(0,0)$
$\frac{1}{4}$ period ←	$\frac{\pi}{2}$	1	→ $(\frac{\pi}{2}, 1)$
$\frac{1}{2}$ period ←	π	0	→ $(\pi, 0)$
$\frac{3}{4}$ period ←	$\frac{3\pi}{2}$	-1	→ $(\frac{3\pi}{2}, -1)$
end ←	2π	0	→ $(2\pi, 0)$

Range: $-1 \leq y \leq 1$

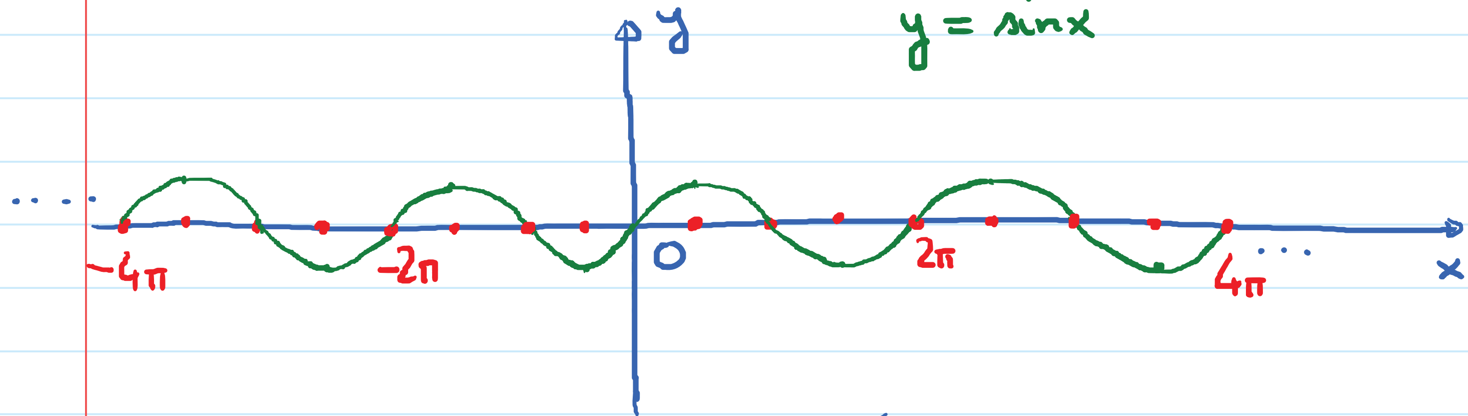


1 period $= 2\pi$



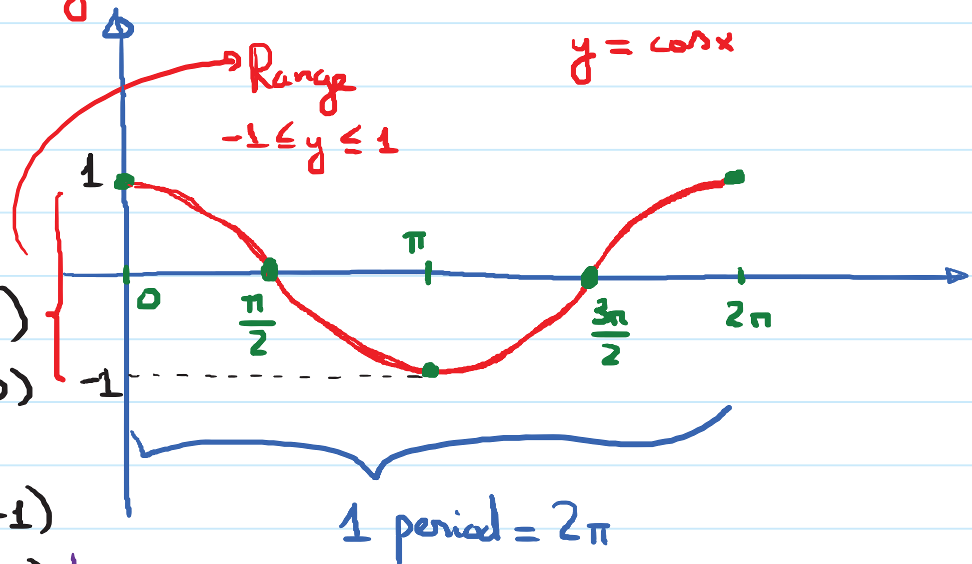
Intercept	Max	Intercept	Min	Intercept
$(0,0)$	$(\frac{\pi}{2}, 1)$	$(\pi, 0)$	$(\frac{3\pi}{2}, -1)$	$(2\pi, 0)$
Start	$\frac{1}{4}$ period	$\frac{1}{2}$ period	$\frac{3}{4}$ period	end

period $= 2\pi$



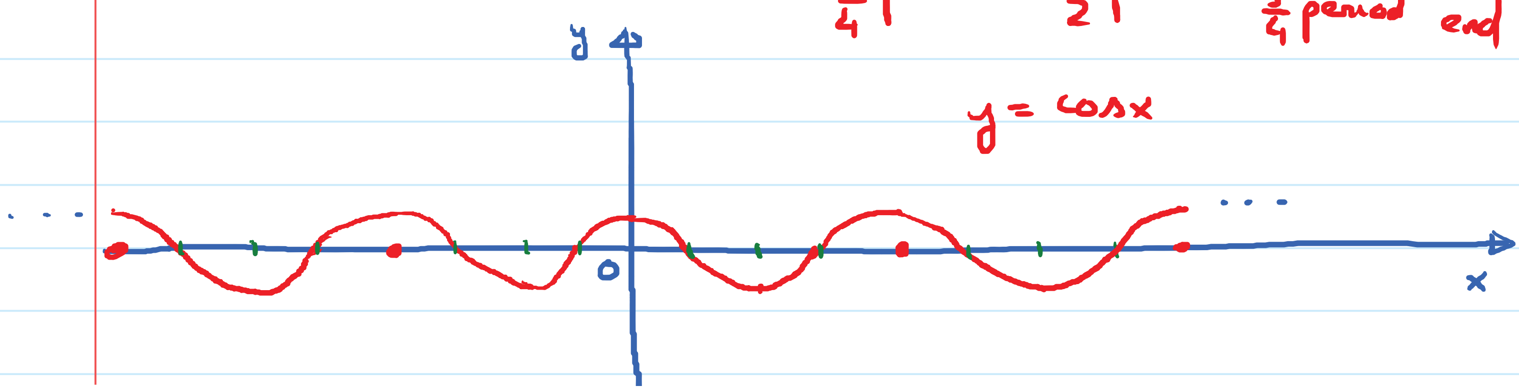
Complete Sine Curve (periodic with period 2π)

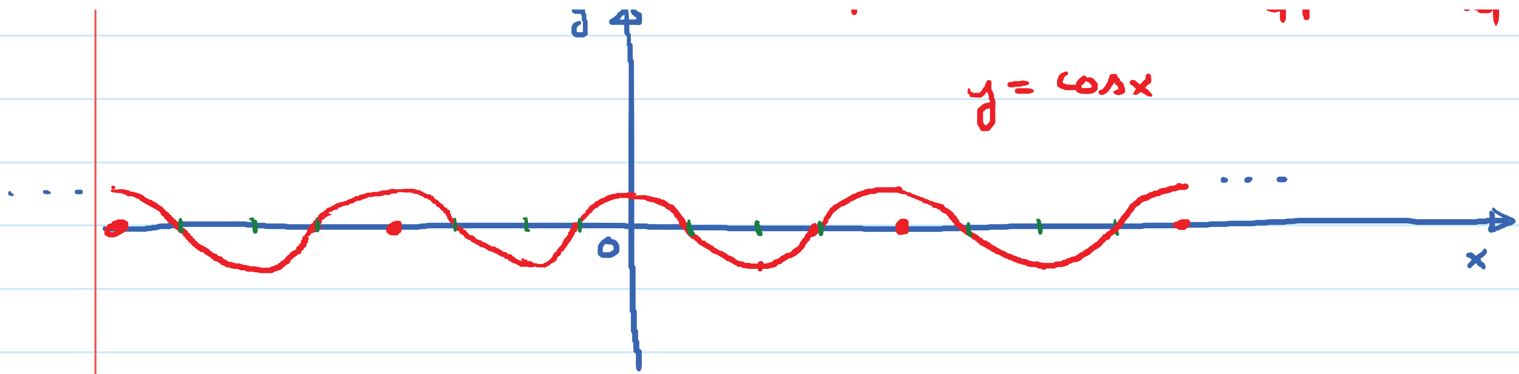
value of trig function → $\boxed{y} = \cos \boxed{x}$ → angle



	x	$y = \cos x$	
Start ←	0	1	→ $(0, 1)$
$\frac{1}{4}$ peri ←	$\frac{\pi}{2}$	0	→ $(\frac{\pi}{2}, 0)$
$\frac{1}{2}$ peri ←	π	-1	→ $(\pi, -1)$
$\frac{3}{4}$ peri ←	$\frac{3\pi}{2}$	0	→ $(\frac{3\pi}{2}, 0)$
end ←	2π	1	→ $(2\pi, 1)$

Max	Intercept	Min	Intercept	Max
$(0, 1)$	$(\frac{\pi}{2}, 0)$	$(\pi, -1)$	$(\frac{3\pi}{2}, 0)$	$(2\pi, 1)$
↓ Start	↓ $\frac{1}{4}$ period	↓ $\frac{1}{2}$ period	↓ $\frac{3}{4}$ period	↓ end





y
y = cosx
0
x

Summary: In 1 period $[0, 2\pi]$

Basic Sine pattern:	Intercept	Max	Intercept	Min	Intercept
	Start	$\frac{1}{4}$ period	$\frac{1}{2}$ period	$\frac{3}{4}$ period	end
Points:	$(0,0)$	$(\frac{\pi}{2}, 1)$	$(\pi, 0)$	$(\frac{3\pi}{2}, -1)$	$(2\pi, 0)$
Basic Cosine pattern:	Max	Intercept	Min	Intercept	Max
	Start	$\frac{1}{4}$ period	$\frac{1}{2}$ period	$\frac{3}{4}$ period	end
points	$(0,1)$	$(\frac{\pi}{2}, 0)$	$(\pi, -1)$	$(\frac{3\pi}{2}, 0)$	$(2\pi, 1)$

2. Graph functions of the form $y = a\sin x$ or $y = a\cos x$. (a is a constant)

Note: $y = a\sin x$, $a\cos x$

Amplitude: is equal to $|a|$

$$\text{Amplitude} = \frac{\text{max value} - \text{min value}}{2}$$

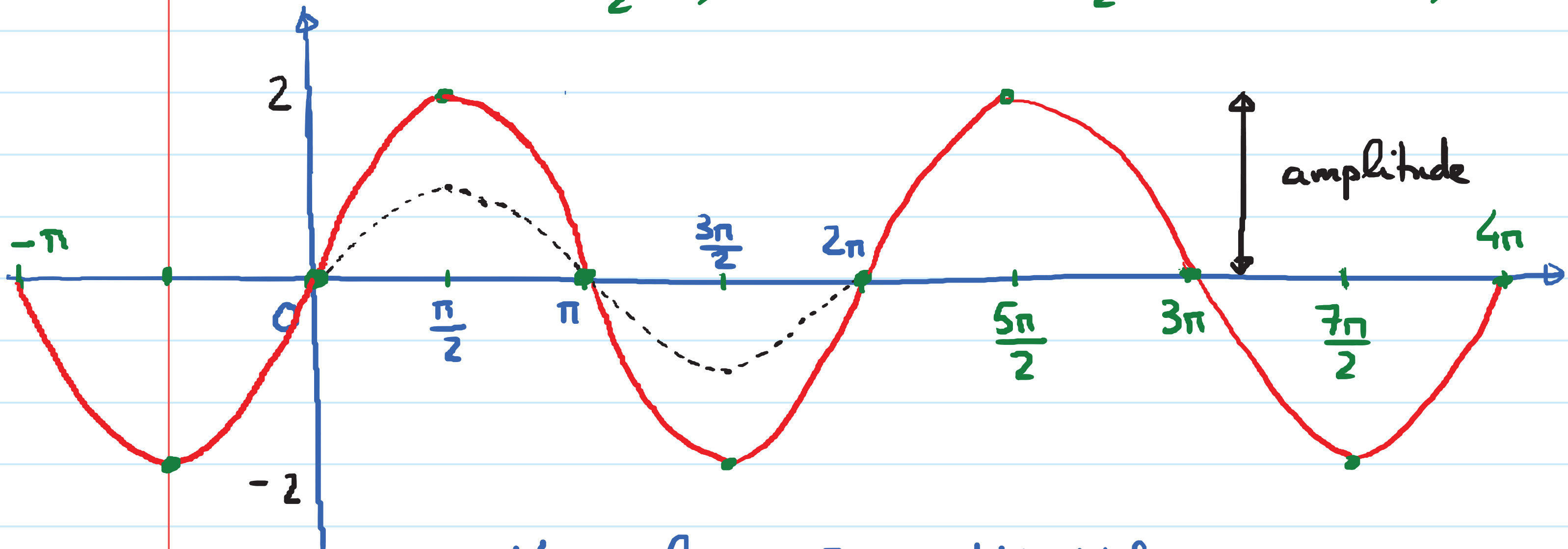
E.g. Sketch the graph of $y = 2\sin x$ on the interval $[-\pi, 4\pi]$.

y of key points

$$y = 2\sin x = 2\boxed{(\sin x)}$$

→ the y-values of the key points will be multiplied by 2

Intercept	Max	Intercept	Min	Intercept
$(0, 0)$	$(\frac{\pi}{2}, 2)$	$(\pi, 0)$	$(\frac{3\pi}{2}, -2)$	$(2\pi, 0)$



Max value = 2 , Min Value = -2

$$\frac{\text{Max} - \text{Min}}{2} = \frac{2-(-2)}{2} = 2 \rightarrow \text{amplitude.}$$

3 Graphs of functions of the form $y = a\sin(bx)$ or $y = a\cos(bx)$ (a, b are constants, assume $b > 0$)

<u>Period</u>: The period of $y = a\sin(bx)$ or $y = a\cos(bx)$

$$\boxed{\text{Period} = \frac{2\pi}{b}}$$

Why? $y = a\sin(\boxed{bx})$ angle

When angle = 0 to angle = 2π → 1 period

$bx = 0 \longrightarrow bx = 2\pi$

$x = 0 \longrightarrow x = \frac{2\pi}{b}$

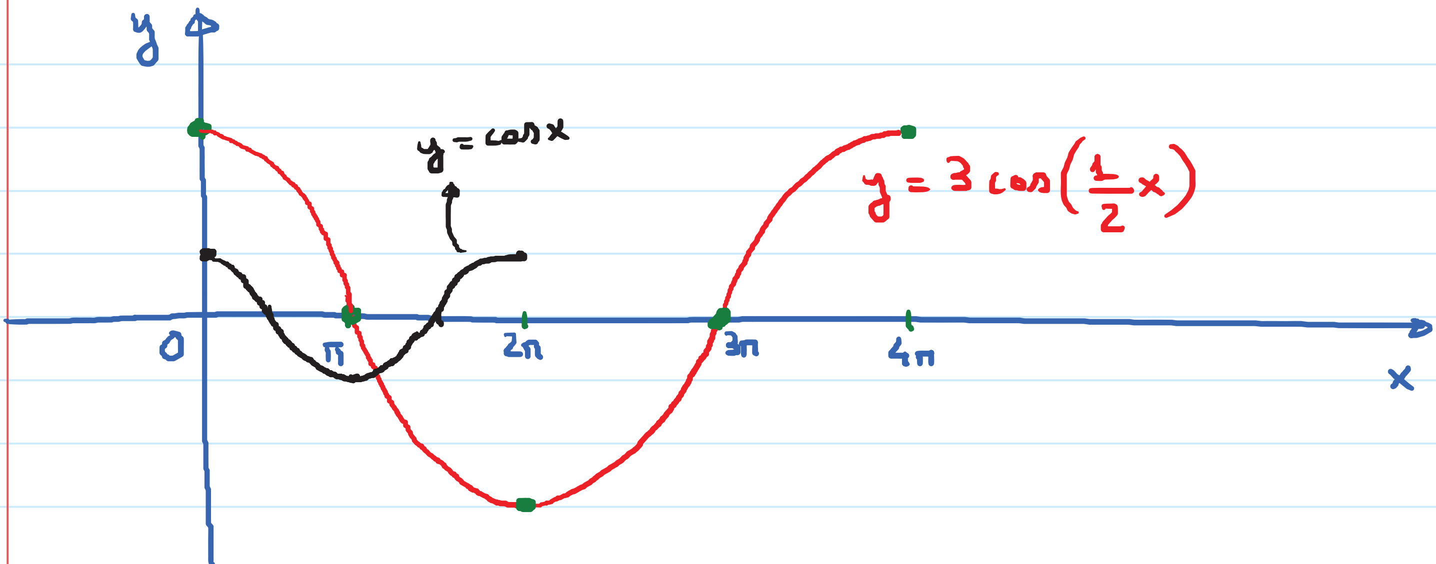
$\left[0, \frac{2\pi}{b}\right]$ → 1 period

E.g. Sketch the graph of $y = 3\cos\left(\frac{1}{2}x\right)$ in 1 period.

$a = 3$; $b = \frac{1}{2}$. Amplitude $= \boxed{3}$

$\text{Period} = \frac{2\pi}{\frac{1}{2}} = \boxed{4\pi}$

Max	Intercept	Min	Intercept	Max
Start	$\frac{1}{4}P$	$\frac{1}{2}P$	$\frac{3}{4}P$.	End
$(0,3)$	$(\pi,0)$	$(2\pi,-3)$	$(3\pi,0)$	$(4\pi,3)$



Ex. Given $y = -\frac{1}{2}\sin\left(\frac{\pi}{2}x\right)$

Find amplitude, period, and 5 Key points in 1 period and use them to sketch the function

$$\text{Amplitude} = |a| = \left|-\frac{1}{2}\right| = \frac{1}{2}$$

$$\text{Period} = \frac{2\pi}{b} = \frac{2\pi}{\frac{\pi}{2}} = 4$$

Intercept	Min	Intercept	Max	Intercept
$(0,0)$	$(1,-\frac{1}{2})$	$(2,0)$	$(3,\frac{1}{2})$	$(4,0)$
Start	$\frac{1}{4}P$	$\frac{1}{2}P$	$\frac{3}{4}P$	end.