4.2. Translations of Sine and Cosine Functions

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Goal: We will learn how to graph functions of the form:

$$y = a\sin(bx - c) + d \quad \text{or}$$

$$y = a\cos(bx - c) + d$$

Reminder:

Basic Sine Curve: $y = \sin x$ → Amplitude $= 1$; Period $= 2\pi$ (Start $= 0$, End $= 2\pi$)

Pattern: Intercept Max Intercept Min Intercept

Intercept	Max	Intercept	Min	Intercept
$(0, 0)$	$(\frac{\pi}{2}, 1)$	$(\pi, 0)$	$(\frac{3\pi}{2}, -1)$	$(2\pi, 0)$
Start	$\frac{1}{4}p$	$\frac{1}{2}p$	$\frac{3}{4}p$	end

Basic Cosine curve:

$y = \cos x$ → Amp. $= 1$; $p = 2\pi$ (Start: 0, End: 2π)

Pattern: Max Intercept Min Intercept Max

Max	Intercept	Min	Intercept	Max
$(0, 1)$	$(\frac{\pi}{2}, 0)$	$(\pi, -1)$	$(\frac{3\pi}{2}, 0)$	$(2\pi, 1)$
Start	$\frac{1}{4}p$	$\frac{1}{2}p$	$\frac{3}{4}p$	end

Variation : $y = a\sin(\boxed{bx})$ or $y = a\cos(\boxed{bx})$

$$\boxed{\text{Amplitude} = |a| \; ; \; \text{Period} = \frac{2\pi}{b} \begin{cases} \text{Start: } 0 \\ \text{End: } \frac{2\pi}{b} \end{cases}}$$

Now, we consider $y = a\sin(bx - c)$ or

$y = a\cos(bx - c)$

$\longrightarrow$ $\boxed{\text{Amplitude} = |a|}$

How to find period and the interval for 1 period?

$y = a\sin(\boxed{bx - c})$ angle

When the angle $bx - c$ goes from 0 to 2π, the sine (or cosine) curve will go through 1 period.

So, to determine the interval for 1 period, we just need to set the angle $bx - c = 0$ and $bx - c = 2\pi$ and solve for x.

$bx - c = 0$; $bx - c = 2\pi$

$\underbrace{x = \frac{c}{b}}_{\text{Start}}$ $x = \frac{c + 2\pi}{b} = \underbrace{\frac{c}{b} + \frac{2\pi}{b}}_{\text{End.}}$

$\longrightarrow$ Interval for 1 period : $\left[\underbrace{\frac{c}{b}}_{\text{Start}}, \underbrace{\frac{c}{b} + \frac{2\pi}{b}}_{\text{end}}\right]$

Length of period $= \frac{2\pi}{b}$ difference $= \frac{2\pi}{b}$

Summary: Characteristics of graphs of

$y = a\sin(bx - c)$ and $y = a\cos(bx - c)$

(1) Amplitude $= |a|$; period $= \frac{2\pi}{b}$ Start : $\frac{c}{b}$ End : $\frac{c}{b} + \frac{2\pi}{b}$

(2) The left and right endpoints of 1 period can be determined by setting $bx - c = 0$ and $bx - c = 2\pi$ and solve for x.

Note: Assume $b > 0$

E.g. Find amplitude, period, left and right endpoints of an interval for 1 period and sketch the graph of

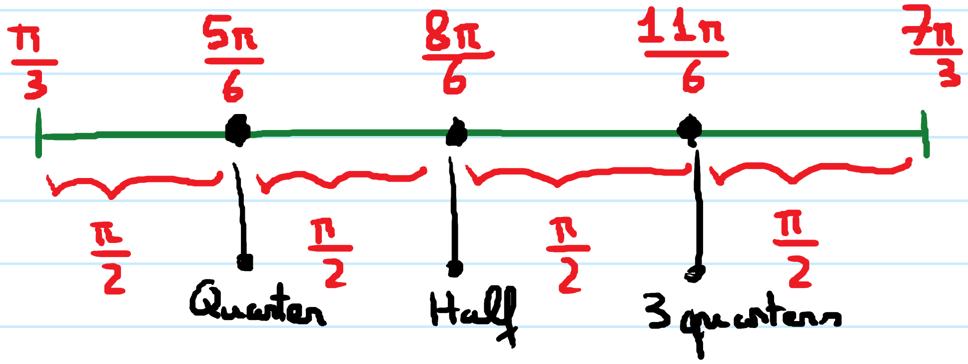
$$y = \frac{1}{2}\sin\left(\boxed{x - \frac{\pi}{3}}\right)$$

Amplitude $= \frac{1}{2}$. Period $= \frac{2\pi}{b} = \frac{2\pi}{1} = 2\pi$.

Left and right endpoints of an interval for 1 period:

$x - \frac{\pi}{3} = 0$; $x - \frac{\pi}{3} = 2\pi$

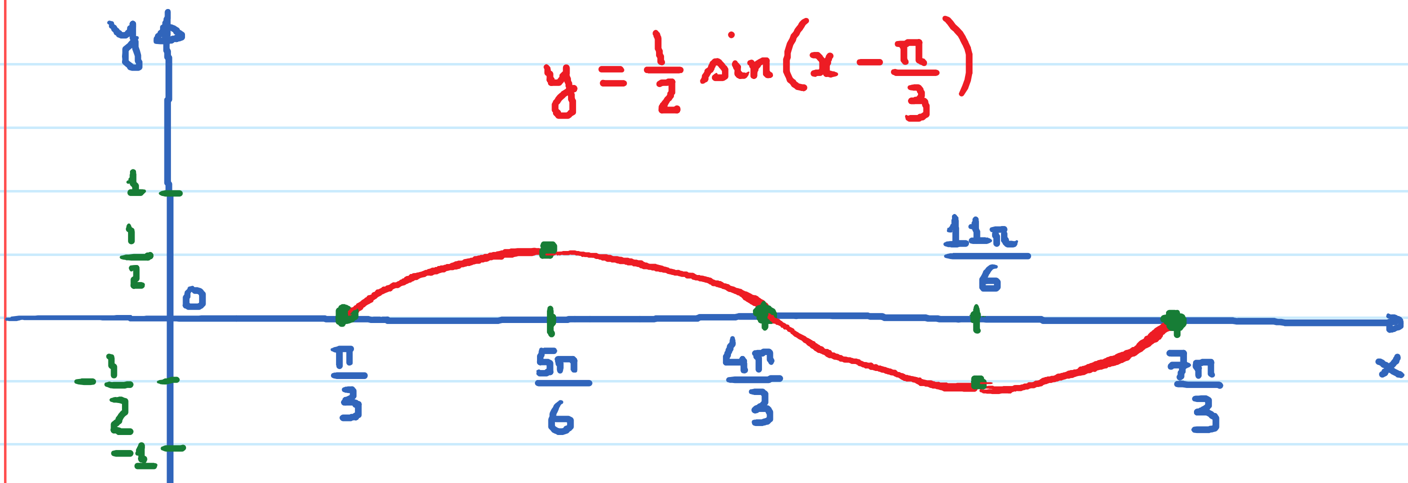
$\underbrace{x = \frac{\pi}{3}}_{\text{Start}}$ $\left(\frac{4\pi}{3}\right)$ $\underbrace{x = \frac{7\pi}{3}}_{\text{end}}$



$(p = 2\pi)$

$\frac{1}{4}p = \frac{\pi}{2}$

Intercept	Max	Intercept	Min	Intercept
$\left(\frac{\pi}{3}, 0\right)$	$\left(\frac{5\pi}{6}, \frac{1}{2}\right)$	$\left(\frac{4\pi}{3}, 0\right)$	$\left(\frac{11\pi}{6}, -\frac{1}{2}\right)$	$\left(\frac{7\pi}{3}, 0\right)$



(Note: graph is shifted to right $\frac{\pi}{3}$ units from basic sine curve)

E.g. Given $y = -3\cos(2\pi x + 4\pi)$

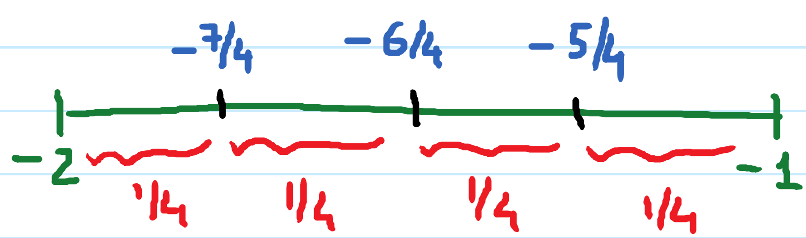
Find amplitude, period, 5 key points in 1 period and sketch a graph of the function.

Amplitude $= |-3| = 3$.

Period $= \frac{2\pi}{2\pi} = 1$.

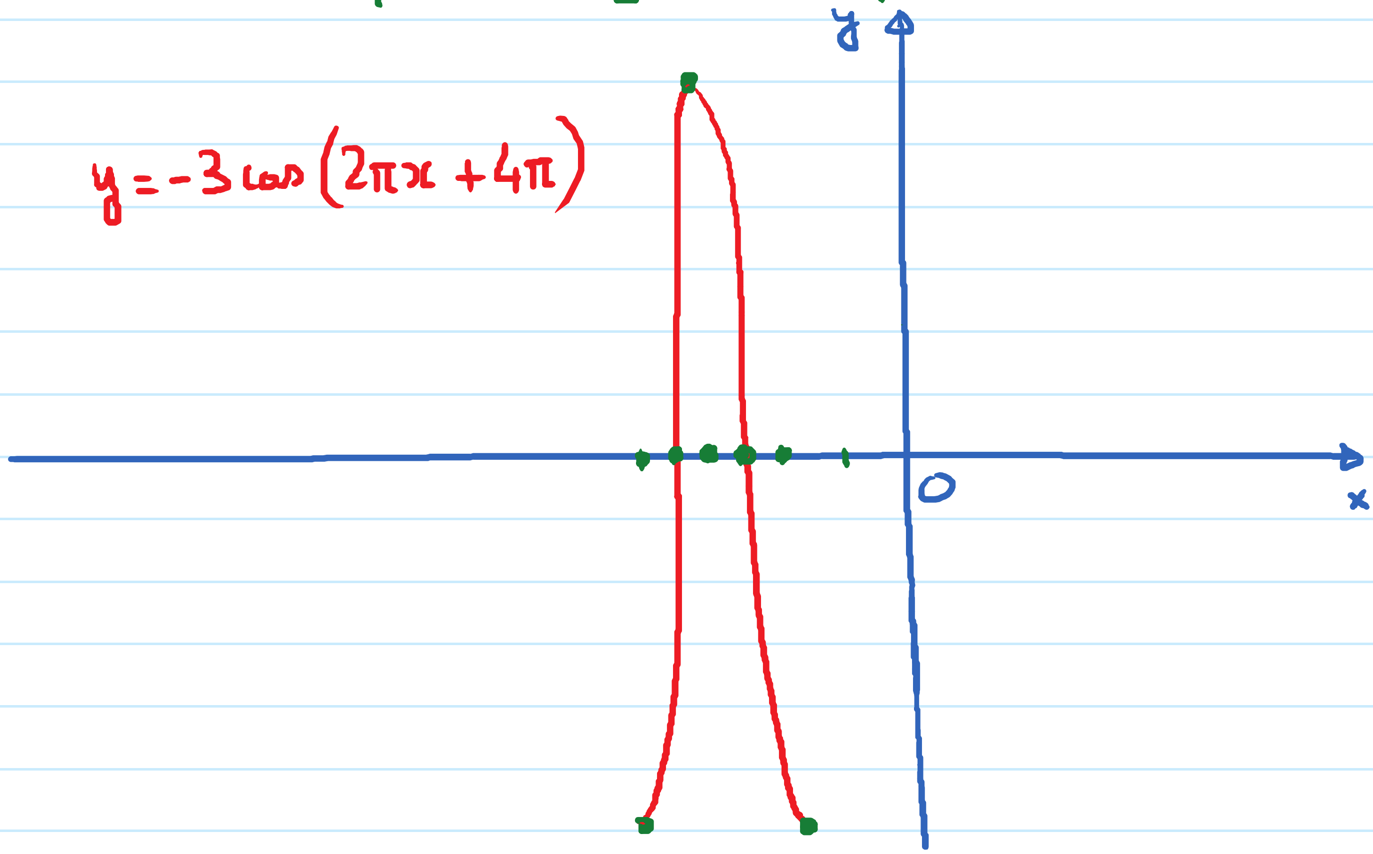
Endpoints: $2\pi x + 4\pi = 0 \rightarrow x = -\frac{4\pi}{2\pi} = -2$.

$2\pi x + 4\pi = 2\pi \rightarrow 2\pi x = -2\pi \rightarrow x = -1$.



$P = 1$

Min	Intercept	Max	Intercept	Min
$(-2,-3)$	$(-\frac{7}{4},0)$	$(-\frac{3}{2},3)$	$(-\frac{5}{4},0)$	$(-1,-3)$



Graphs of $y = a\sin(bx-c)+d$ or

$$y = a\cos(bx-c)+d.$$

These can be obtained from $y = a\sin(bx-c)$ or $y = a\cos(bx-c)$ by shifting the latter d units $\boxed{\text{up if } d>0}$ or $\boxed{\text{down if } d<0}$

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E.g. Find amplitude, period, 5 key points of

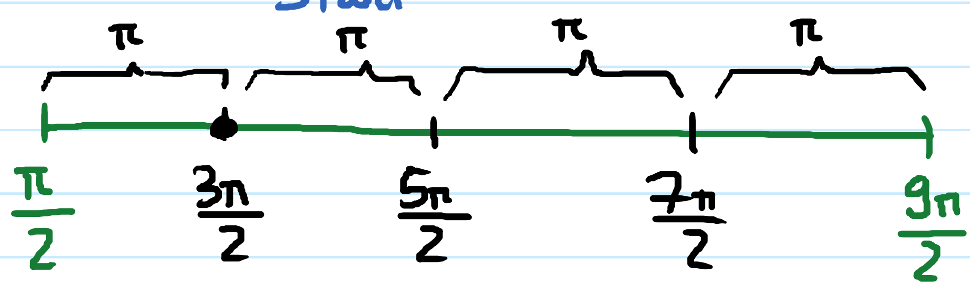
$$y = -3 + 2\sin\left(\frac{1}{2}x - \frac{\pi}{4}\right)$$

Amplitude $= 2$; period $= \frac{2\pi}{b} = \frac{2\pi}{\frac{1}{2}} = 4\pi$.

Endpoints: $\frac{1}{2}x - \frac{\pi}{4} = 0$; $\frac{1}{2}x - \frac{\pi}{4} = 2\pi$

$\frac{1}{2}x = \frac{\pi}{4}$; $\frac{1}{2}x = \frac{9\pi}{4}$

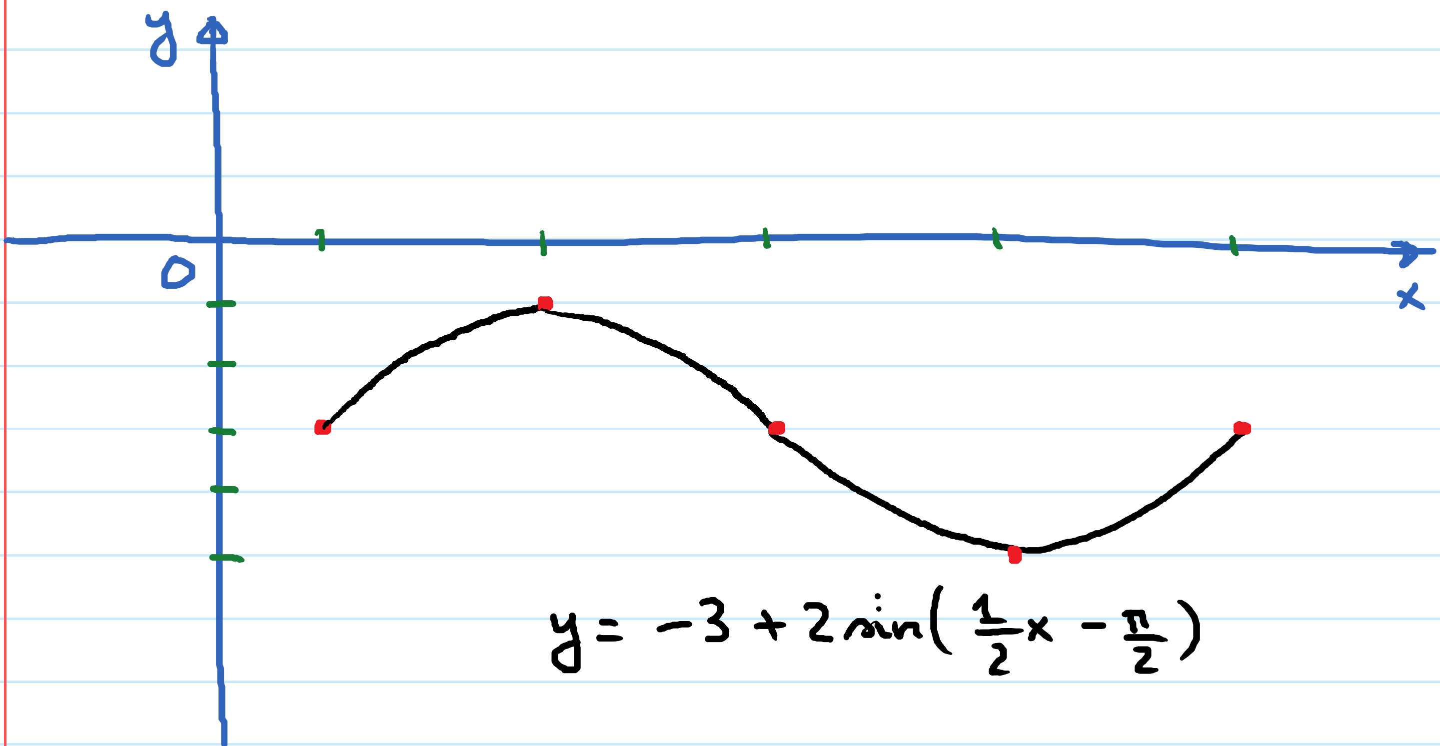
$\underbrace{x = \frac{\pi}{2}}_{\text{Start}}$; $\underbrace{x = \frac{9\pi}{2}}_{\text{End.}}$



$(p = 4\pi)$

"Intercept" Max "Intercept" Min "Intercept"

$\left(\frac{\pi}{2}, -3\right)$ $\left(\frac{3\pi}{2}, -1\right)$ $\left(\frac{5\pi}{2}, -3\right)$ $\left(\frac{7\pi}{2}, -5\right)$ $\left(\frac{9\pi}{2}, -3\right)$



$$y = -3 + 2\sin\left(\frac{1}{2}x - \frac{\pi}{2}\right)$$