4.3. and 4.4. Graphs of the Remaining Trig Functions

Tuesday, March 5, 2019 8:04 AM

(1) The tangent function.

Basic tangent function : $\boxed{y = \tan x}$

$$\tan x = \frac{\sin x}{\cos x}$$

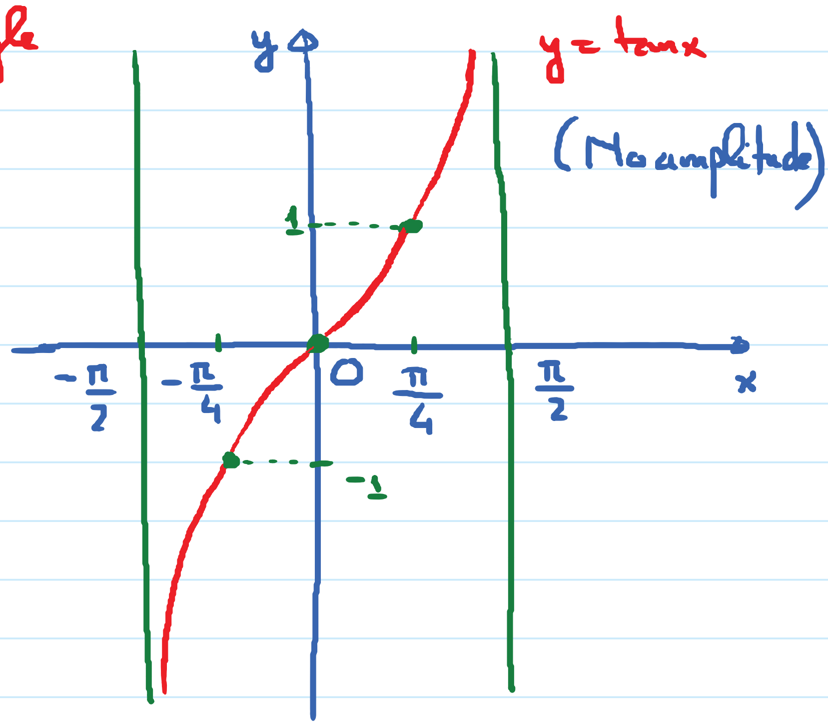
The tangent is undefined when $\cos x = 0$.

$\cos x = 0$ when $x = \frac{\pi}{2}, \frac{3\pi}{2}, \frac{5\pi}{2}, \frac{7\pi}{2}, \ldots$

$-\frac{\pi}{2}, -\frac{3\pi}{2}, \ldots$

Period of the tangent is π.

x	$\boxed{y} = \tan \boxed{x}$ ← angle
$-\frac{\pi}{2}$	undefined
$-\frac{\pi}{4}$	-1
0	0
$\frac{\pi}{4}$	1
$\frac{\pi}{2}$	undefined



Pattern : V.A. Below Intercept Above V.A

Start $\frac{1}{4}P$ $\frac{1}{2}P$ $\frac{3}{4}P$ End

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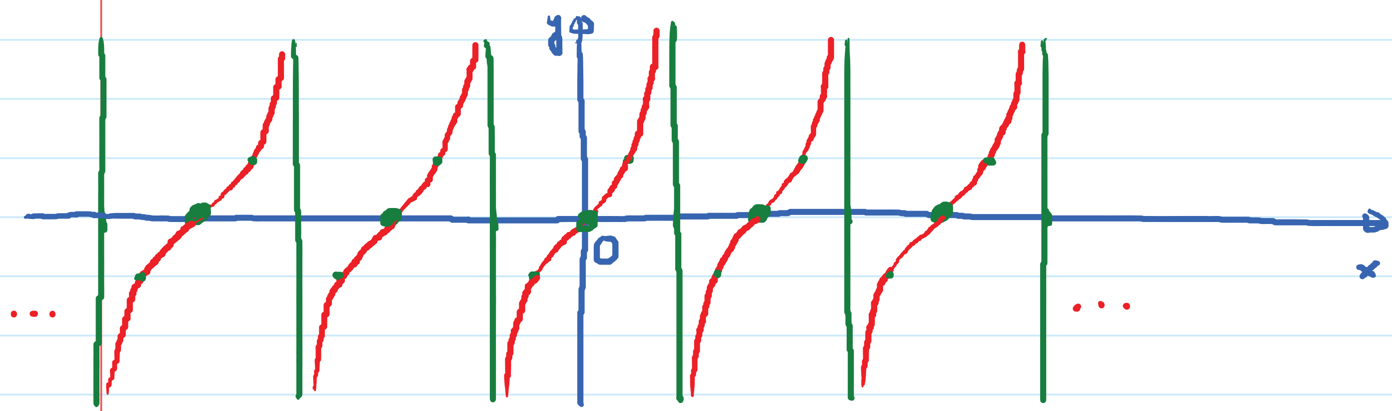

Complete graph of $y = \tan x$.

No amplitude. Period $= \pi$.

Domain: All real #s except $x = \pm\frac{\pi}{2}, \pm\frac{3\pi}{2}, \pm\frac{5\pi}{2}, \ldots$

Range: $(-\infty, \infty)$

Process for graphing variations

$$y = \boxed{a}\tan\left(\boxed{bx - c}\right) + d$$

Stretch/Shrink

up/down shift

To find endpoints of 1 cycle:

Set $bx - c = -\frac{\pi}{2}$ and $bx - c = \frac{\pi}{2}$ and solve for x

Period $= \frac{\pi}{b}$. Use a and d to adjust y-values of key points.

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(2) The cotangent Function.

Basic: $y = \cot x$.

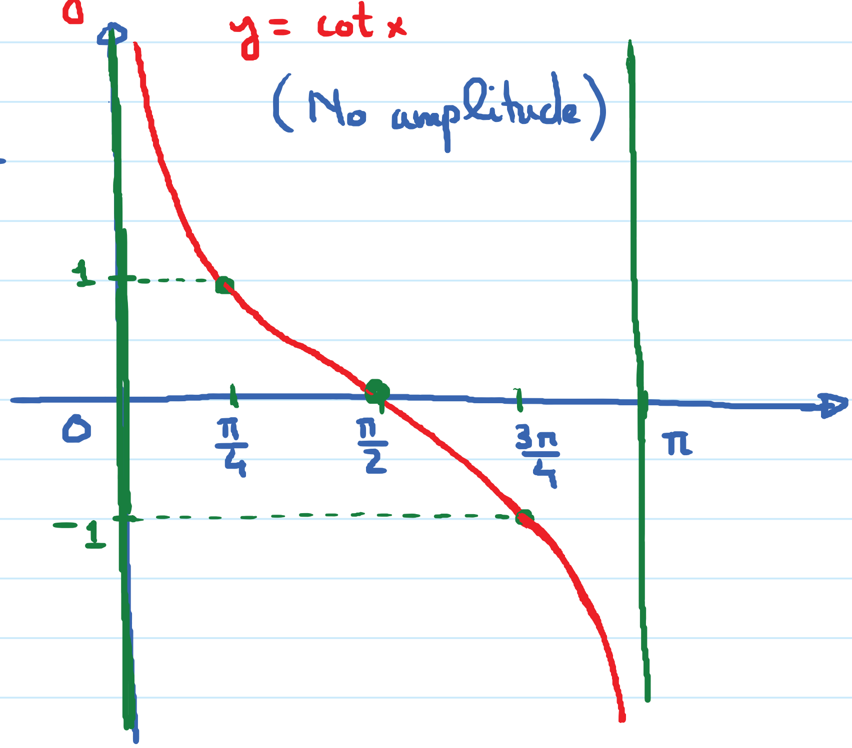
$$\cot x = \frac{\cos x}{\sin x}$$

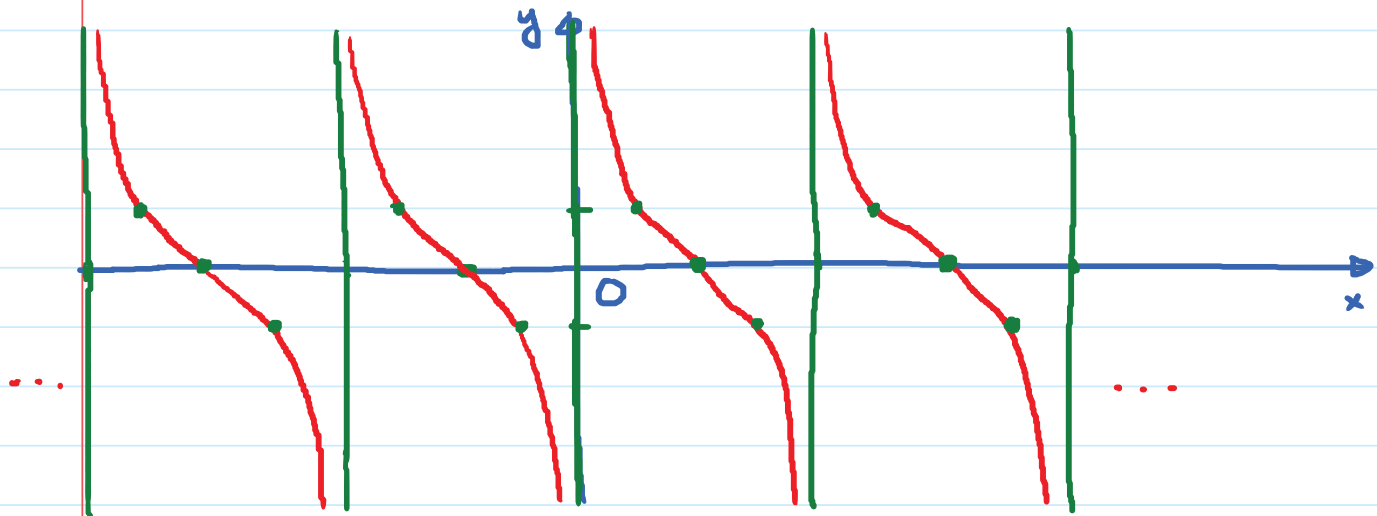
Cotangent is undefined when $\sin x = 0$

$\sin x = 0$ when $x = 0, \pi, 2\pi, 3\pi, 4\pi, \ldots$
$-\pi, -2\pi, -3\pi, -4\pi, \ldots$

Period $= \pi$

x	$\boxed{y} = \cot \boxed{x}$ (angle)
0	undefined
$\frac{\pi}{4}$	1
$\frac{\pi}{2}$	0
$\frac{3\pi}{4}$	-1
π	undefined





$y = \cot x$

No amplitude, period $= \pi$.

Domain: All real except $x = 0, \pm\pi, \pm 2\pi, \pm 3\pi, \ldots$

Range: $(-\infty, \infty)$

Process for graphing variations

$$y = \boxed{a} \cot(\boxed{bx - c}) + d$$

a: Stretch/Shrink

+d: up/down shift

Endpoints of 1 cycle: set $bx - c = 0$ and $bx - c = \pi$ and solve for x.

Period $= \dfrac{\pi}{b}$.

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3 The cosecant function

Basic: $y = \csc x$

$$\csc x = \frac{1}{\sin x}$$

$\csc x$ is undefined when $x = 0, \pm\pi, \pm 2\pi, \pm 3\pi, \ldots$

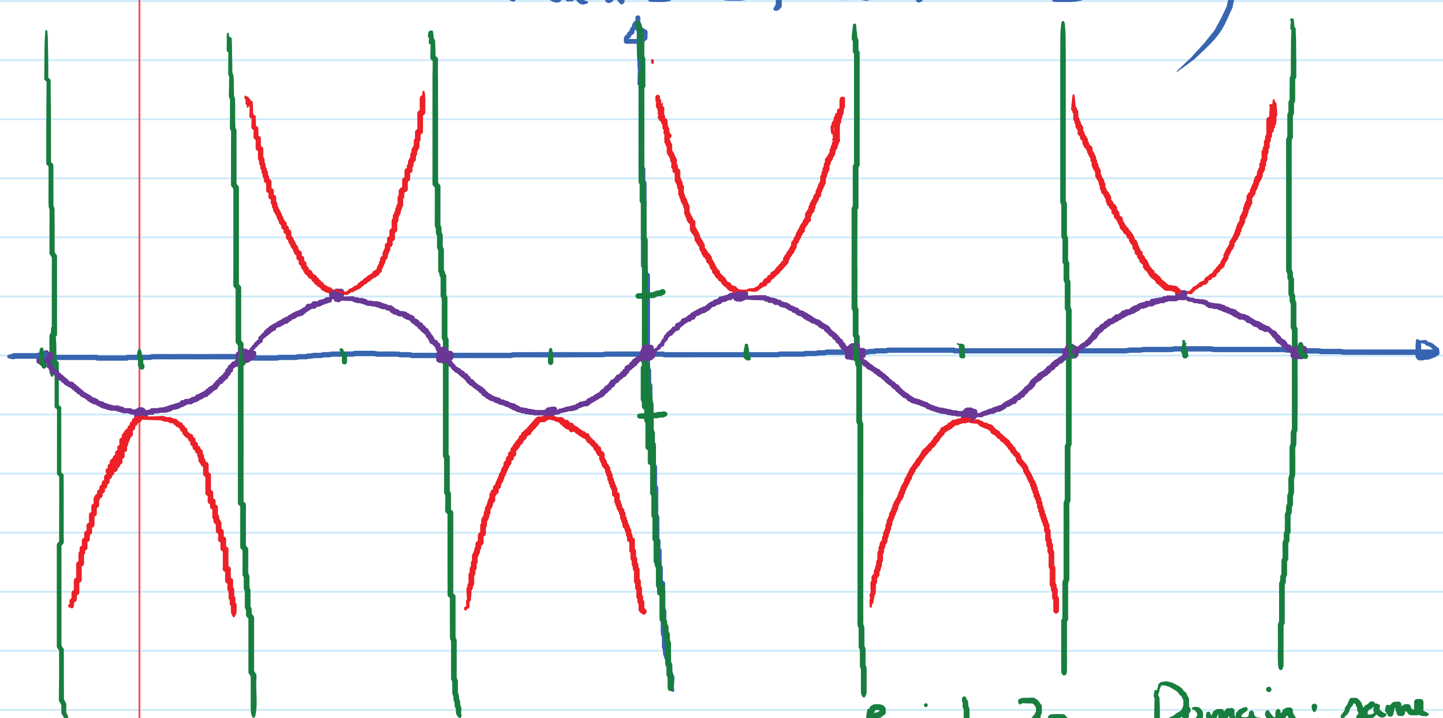
Period of $y = \csc x$ is 2π.

To graph $y = \csc x$, we first graph $y = \sin x$.

(Note: when $\sin x = 0$, $y = \csc x$ has V.A.

when $\sin x = 1$; $\csc x = 1$

$\sin x = -1$; $\csc x = -1$)



Range: $(-\infty, -1] \cup [1, \infty)$

$y = \csc x$

Period $= 2\pi$

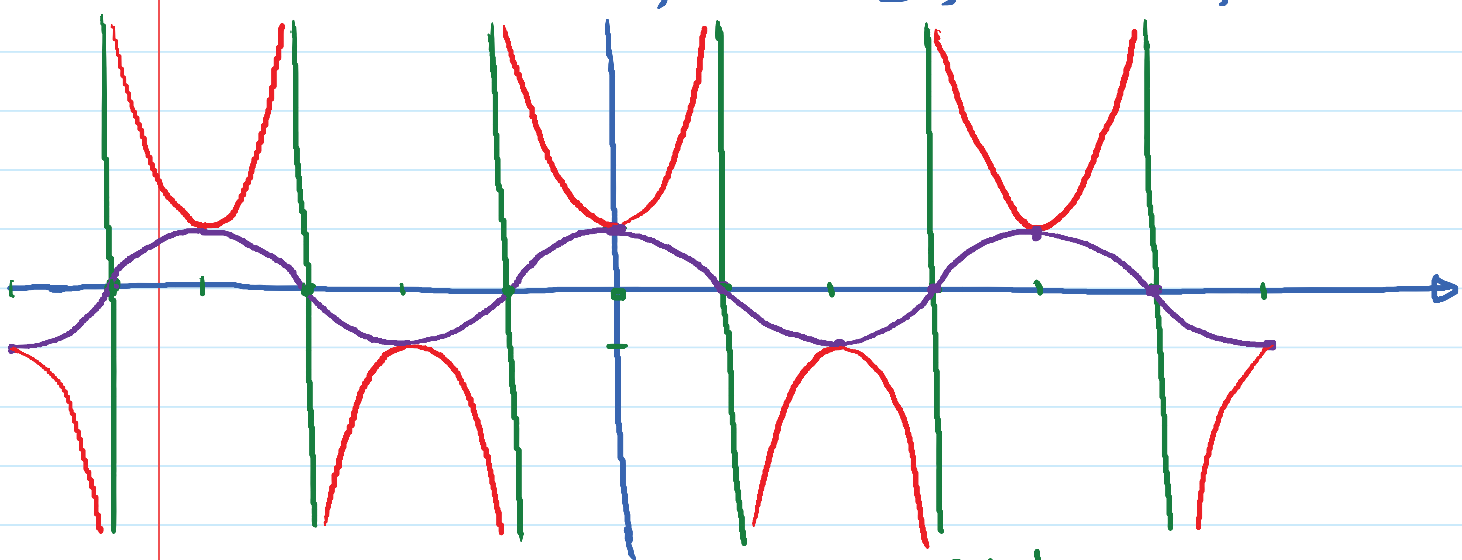
No amplitude

Domain: same as domain of cot

④ Graph of secant . $\left(y = \sec x = \frac{1}{\cos x}\right)$

Sketch $y = \cos x$ first, when cosine has intercepts secant will have V.A.

When $\cos x = 1$, $\sec x = 1$; $\cos x = -1$, $\sec x = -1$.



$y = \sec x$ — No amplitude; Period $= 2\pi$

Domain: all real except $x = \pm\frac{\pi}{2}, \pm\frac{3\pi}{2}, \ldots$

Range: $(-\infty, -1] \cup [1, \infty)$

Process for graphing variations:

$y = a\sec(bx - c) + d$ or $y = a\csc(bx - c) + d$

We just need to graph $y = a\cos(bx - c) + d$ or

$y = a\sin(bx - c) + d$ first and use these graphs.