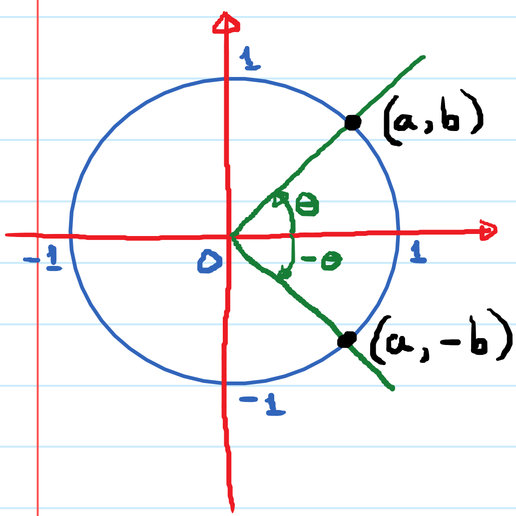
5.1 Fundamental Identities

Even / odd Identities



$\cos(\theta) = \boxed{a}$; $\sin(\theta) = \boxed{b}$

$\cos(-\theta) = \boxed{a}$; $\sin(-\theta) = \boxed{-b}$

For any angle θ, we have :

$$\boxed{\cos(-\theta) = \cos(\theta) \; ; \; \sin(-\theta) = -\sin(\theta)}$$

(Even/odd Identities)

E.g. $\cos\left(-\frac{\pi}{4}\right) = \cos\left(\frac{\pi}{4}\right) = \frac{\sqrt{2}}{2}$.

$\sin\left(-\frac{5\pi}{6}\right) = -\sin\left(\frac{5\pi}{6}\right) = -\frac{1}{2}$.

$$\tan(-\theta) = \frac{\sin(-\theta)}{\cos(-\theta)} = \frac{-\sin(\theta)}{\cos(\theta)} = -\underbrace{\boxed{\frac{\sin\theta}{\cos\theta}}}_{\tan\theta}$$

$\tan(-\theta) = -\tan\theta$.

$\cot(-\theta) = -\cot\theta$

$$\underbrace{\sec(-\theta)} = \frac{1}{\cos(-\theta)} = \frac{1}{\cos(\theta)} = \underbrace{\sec(\theta)}$$

$$\csc(-\theta) = -\csc(\theta).$$

To sum up, for any angle θ

Even odd Identities

$$\boxed{\begin{array}{ll} \sin(-\theta) = -\sin\theta & ; \cos(-\theta) = \cos\theta \\ \csc(-\theta) = -\csc\theta & ; \sec(-\theta) = \sec\theta \\ \tan(-\theta) = -\tan\theta & ; \cot(-\theta) = -\cot\theta \end{array}}$$

Change the name of the angle θ to x.

$$\boxed{\begin{array}{ll} \sin(-x) = -\sin x & ; \cos(-x) = \cos(x) \\ \csc(-x) = -\csc x & ; \sec(-x) = \sec(x) \\ \tan(-x) = -\tan(x); & \cot(-x) = -\cot(x) \end{array}}$$

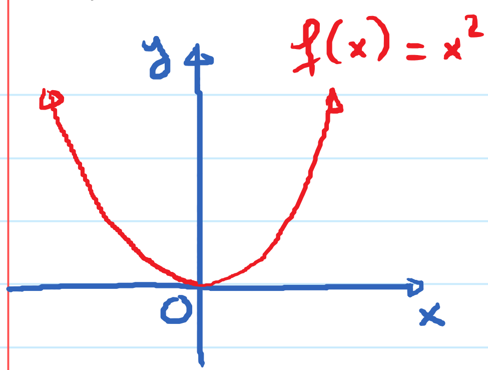
sine, cosecant, tangent, cotangent are ODD functions

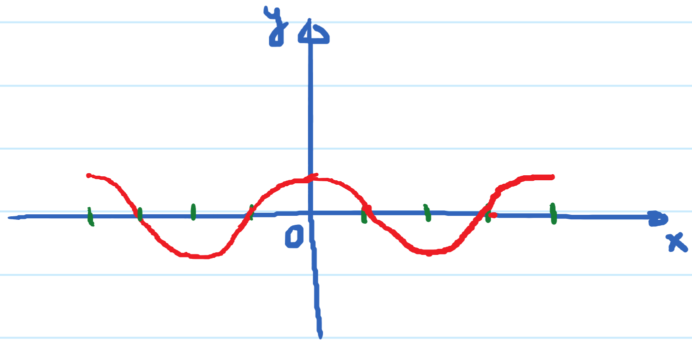
cosine, secant are EVEN functions

Recall (college algebra):

E.g. $f(x) = x^2$ is an even function.

$$f(-x) = (-x)^2 = x^2 \rightarrow f(-x) = f(x)$$

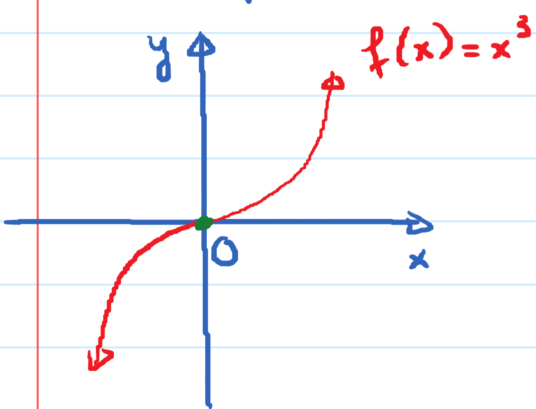


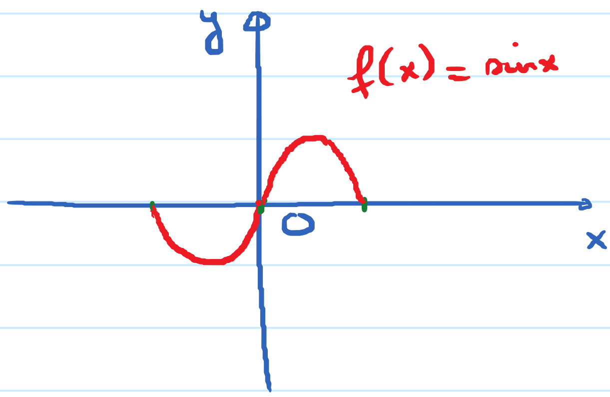


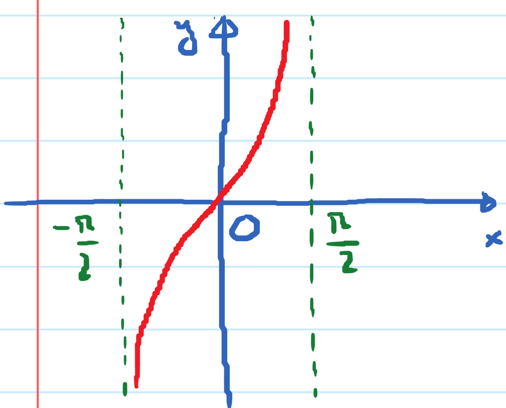
Graph of any even function is symmetric w.r.t y-axis

E.g. $f(x) = x^3$ is an odd function.

$$f(-x) = (-x)^3 = -x^3 = -f(x)$$



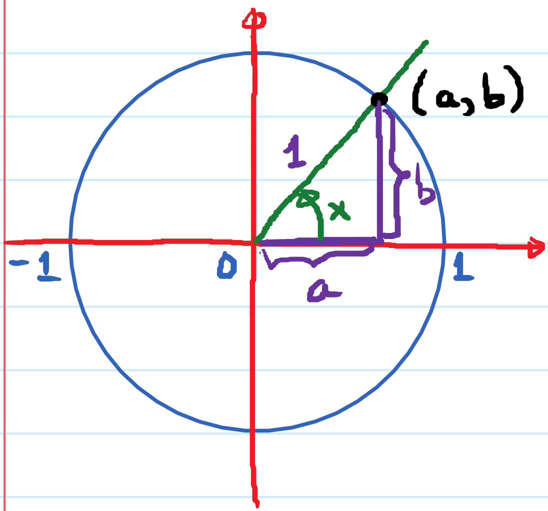




Graph of any odd function is symmetric w.r.t. origin

Pythagorean Identities. For any angle x

$$\sin^2(x) + \cos^2(x) = 1$$



$\sin(x) = b$; $\cos(x) = a$

$$\sin^2(x) + \cos^2(x) = b^2 + a^2$$
$$= 1$$

I $\boxed{\sin^2(x) + \cos^2(x) = 1}$

$\boxed{\sin^2(x) = 1 - \cos^2(x)}$

$\boxed{\cos^2(x) = 1 - \sin^2(x)}$

II $\boxed{\tan^2(x) + 1 = \sec^2(x)}$

$\boxed{\tan^2(x) = \sec^2(x) - 1}$

$\boxed{\sec^2(x) - \tan^2(x) = 1}$

III $\boxed{\cot^2(x) + 1 = \csc^2(x)}$

$\boxed{\cot^2(x) = \csc^2(x) - 1}$

$\boxed{\csc^2(x) - \cot^2(x) = 1}$

Reciprocal Identities.

$$\boxed{\sec(x) = \frac{1}{\cos(x)} \; ; \; \csc(x) = \frac{1}{\sin(x)} \qquad \tan(x) = \frac{1}{\cot(x)} \; ; \; \cot(x) = \frac{1}{\tan(x)}}$$

Quotient Identities

$$\boxed{\tan(x) = \frac{\sin(x)}{\cos(x)} \; ; \; \cot(x) = \frac{\cos(x)}{\sin(x)}}$$

E.x. Is it true that for any angle x, we have:

$$[\sec(x) - \tan(x)] \cdot [\sec(x) + \tan(x)] = 1 \; ?$$

Left Hand side =

$$\sec^2(x) + \cancel{\sec(x)\tan(x)} - \cancel{\tan(x)\cdot\sec(x)} - \tan^2(x)$$

$$= \sec^2(x) - \tan^2(x) = 1$$

E.x. Is it true that

$$\left[\sin(x) + \cos(x)\right]^2 = 1 + 2\sin(x)\cdot\cos(x)$$

for any angle x?

$$LHS = \left[\sin(x) + \cos(x)\right]\cdot\left[\sin(x) + \cos(x)\right]$$

$$= \sin^2(x) + \underbrace{\sin(x)\cos(x) + \cos(x)\sin(x)} + \cos^2(x)$$

$$= \underbrace{\sin^2(x) + \cos^2(x)} + 2\sin(x)\cos(x)$$

$$= 1 + 2\sin(x)\cos(x) = RHS$$

Ex. Is it true that

$$\sin^4 x - \cos^4 x = \sin^2 x - \cos^2 x$$ for any angle x