5.2. Verify Trig Identities

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Reminder: $\sin(-x) = -\sin(x)$; $\cos(-x) = \cos(x)$

Pythagorean: $\sin^2(x) + \cos^2(x) = 1$

$$\tan^2(x) + 1 = \sec^2(x)$$

$$\cot^2(x) + 1 = \csc^2(x)$$

Reciprocal: $\sec(x) = \dfrac{1}{\cos(x)}$; $\csc(x) = \dfrac{1}{\sin(x)}$

Quotient: $\tan(x) = \dfrac{\sin(x)}{\cos(x)}$; $\cot(x) = \dfrac{\cos(x)}{\sin(x)}$

Trig Identity

A trig identity is an equation that holds for any angle x (or angle θ)

How to verify a trig identity

LHS = RHS.

* Pick a side one side of the equation to work with.
* Use the fundamental identities above or algebraic identities / operations (distribute, factor, combine like

terms, add/subtract/multiply/divide fractions) to change that side until you get the same expression as the other side.

E.g. Verify the given identity.

(1) $$\frac{\cot(x)}{\csc(x)} = \cos(x)$$

divide fractions

$$\text{LHS} = \frac{\cot(x)}{\csc(x)} = \frac{\boxed{\frac{\cos(x)}{\sin(x)}}}{\boxed{\frac{1}{\sin(x)}}} = \frac{\cos(x)}{\sin(x)} \cdot \frac{\sin(x)}{1}$$

quotient, reciprocal

mult. fraction

cancel common factor

$$= \frac{\cos(x)\cdot \cancel{\sin(x)}}{\cancel{\sin(x)}\cdot 1} = \cos(x) = \text{RHS}.$$

(Done!)

(2) $$\cot(x) + \tan(x) = \sec(x)\cdot\csc(x)$$

$$\text{RHS} = \sec(x)\cdot\csc(x) = \frac{1}{\cos(x)}\cdot\frac{1}{\sin(x)} = \frac{1}{\cos(x)\cdot\sin(x)}$$

Pythagorean identity

$$= \frac{\sin^2(x) + \cos^2(x)}{\cos(x)\cdot\sin(x)}$$

$$= \frac{\sin^{\cancel{2}}(x)}{\cos(x)\cdot\cancel{\sin(x)}} + \frac{\cos^{\cancel{2}}(x)}{\cancel{\cos(x)}\cdot\sin(x)}$$

$$= \frac{\sin(x)}{\cos(x)} + \frac{\cos(x)}{\sin(x)} = \tan(x) + \cot(x)$$

Quotient

$= \text{LHS}$ ▨

③ $\sin^4(x) - \cos^4(x) = 2\sin^2(x) - 1.$

Difference between Squares Factorization Formula:

$$\boxed{A^2 - B^2 = (A+B)(A-B)}$$

$$\text{LHS} = \sin^4(x) - \cos^4(x) = \big(\underbrace{\boxed{\sin^2(x)}}_{A}\big)^2 - \big(\underbrace{\boxed{\cos^2(x)}}_{B}\big)^2$$

$$= \big(\underbrace{\sin^2(x) + \cos^2(x)}_{1}\big)\cdot\big(\sin^2(x) - \cos^2(x)\big)$$

Pythagorean Identity

$$= \sin^2(x) - \cos^2(x)$$

$$= \sin^2(x) - \big(1 - \sin^2(x)\big)$$

$$= \sin^2(x) - 1 + \sin^2(x)$$

$$= 2\sin^2(x) - 1 = \text{RHS}$$ ▨

(4) $$\frac{1}{1-\sin(x)} + \frac{1}{1+\sin(x)} = 2\sec^2(x)$$

$$LHS = \frac{1\cdot(1+\sin(x))}{(1-\sin(x))(1+\sin(x))} + \frac{1\cdot(1-\sin(x))}{(1+\sin(x)\cdot)(1-\sin(x))}$$

$$= \frac{1+\sin(x)}{(1-\sin(x))\cdot(1+\sin(x))} + \frac{1-\sin(x)}{(1+\sin(x))(1-\sin(x))}$$

$$= \frac{1+\cancel{\sin(x)}+1-\cancel{\sin(x)}}{(1-\sin(x))(1+\sin(x))}$$

$$= \frac{2}{(1-\sin(x))(1+\sin(x))}$$

$$= \frac{2}{1-\sin^2(x)}$$

$$= \frac{2}{\cos^2(x)} = 2\sec^2(x) = RHS \quad \boxed{\checkmark}$$

(5) $$\frac{\cot(x)+1}{\cot(x)-1} = \frac{1+\tan(x)}{1-\tan(x)}$$

$$LHS = \frac{\frac{\cos(x)}{\sin(x)} + \frac{1\cdot\sin(x)}{1\cdot\sin(x)}}{\frac{\cos(x)}{\sin(x)} - \frac{1\cdot\sin(x)}{1\cdot\sin(x)}} = \frac{\frac{\cos(x)}{\sin(x)} + \frac{\sin(x)}{\sin(x)}}{\frac{\cos(x)}{\sin(x)} - \frac{\sin(x)}{\sin(x)}}$$

$$= \frac{\boxed{\dfrac{\cos(x)+\sin(x)}{\sin(x)}}}{\boxed{\dfrac{\cos(x)-\sin(x)}{\sin(x)}}}$$

$$= \frac{\cos(x)+\sin(x)}{\cancel{\sin(x)}} \cdot \frac{\cancel{\sin(x)}}{\cos(x)-\sin(x)}$$

$$= \boxed{\frac{\cos(x)+\sin(x)}{\cos(x)-\sin(x)}}$$

$$\text{RHS} = \frac{1+\tan(x)}{1-\tan(x)} = \frac{\dfrac{1\cdot\cos x}{1\cdot\cos x} + \dfrac{\sin(x)}{\cos(x)}}{\dfrac{1\cdot\cos x}{\cos x} - \dfrac{\sin(x)}{\cos(x)}}$$

$$= \frac{\dfrac{\cos(x)+\sin(x)}{\cos(x)}}{\dfrac{\cos(x)-\sin(x)}{\cos(x)}} = \frac{\cos(x)+\sin(x)}{\cancel{\cos(x)}} \cdot \frac{\cancel{\cos(x)}}{\cos(x)-\sin(x)}$$

$$= \boxed{\frac{\cos(x)+\sin(x)}{\cos(x)-\sin(x)}}$$

So, LHS = RHS (b/c both $= \dfrac{\cos(x)+\sin(x)}{\cos(x)-\sin(x)}$)