5.3. Sum and Difference Identities for Cosine.

Tuesday, March 26, 2019 8:08 AM

* Idea:

$$\cos(60^\circ) = \frac{1}{2}$$

$$\cos(45^\circ) = \frac{\sqrt{2}}{2}$$

$$\rightarrow \cancel{\cos(105^\circ) = \frac{1}{2} + \frac{\sqrt{2}}{2}}$$

$$\cancel{\cos(15^\circ) = \frac{1}{2} - \frac{\sqrt{2}}{2}}$$

Incorrect

→ Identities for finding the cosine of a sum and a difference:

A, B : any 2 angles.

$$\boxed{\begin{aligned} \cos(A+B) &= \cos(A)\cdot\cos(B) - \sin(A)\cdot\sin(B) \\ \cos(A-B) &= \cos(A)\cdot\cos(B) + \sin(A)\cdot\sin(B) \end{aligned}}$$

E.g. $\cos(105^\circ) = \cos(60^\circ + 45^\circ)$

$$= \cos(60^\circ)\cdot\cos(45^\circ) - \sin(60^\circ)\cdot\sin(45^\circ)$$

$$= \frac{1}{2}\cdot\frac{\sqrt{2}}{2} - \frac{\sqrt{3}}{2}\cdot\frac{\sqrt{2}}{2} = \frac{\sqrt{2}}{4} - \frac{\sqrt{6}}{4}$$

$$\boxed{\cos(105^\circ) = \frac{\sqrt{2} - \sqrt{6}}{4}}$$

$$\cos(15^\circ) = \cos(60^\circ - 45^\circ)$$

$$= \cos(60^\circ)\cos(45^\circ) + \sin(60^\circ)\sin(45^\circ)$$

$$= \frac{1}{2}\cdot\frac{\sqrt{2}}{2} + \frac{\sqrt{3}}{2}\cdot\frac{\sqrt{2}}{2} = \frac{\sqrt{2}}{4} + \frac{\sqrt{6}}{4}$$

$$\cos(15^\circ) = \frac{\sqrt{2}+\sqrt{6}}{4}$$

E.g. Find exact value of the given expression.

(1) $\cos(195^\circ)$ (2) $\cos\left(-\frac{\pi}{12}\right)$

(3) $\cos(173^\circ)\cos(128^\circ) + \sin(173^\circ)\sin(128^\circ)$

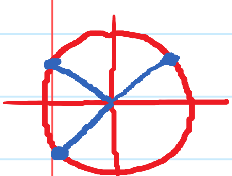
(4) $\cos\left(\frac{\pi}{18}\right)\cos\left(\frac{\pi}{9}\right) - \sin\left(\frac{\pi}{18}\right)\sin\left(\frac{\pi}{9}\right)$

Sol:

(1) $\cos(195^\circ) = \cos(135^\circ + 60^\circ)$

$$= \cos(135^\circ)\cdot\cos(60^\circ) - \sin(135^\circ)\cdot\sin(60^\circ)$$

$$= -\frac{\sqrt{2}}{2}\cdot\frac{1}{2} - \frac{\sqrt{2}}{2}\cdot\frac{\sqrt{3}}{2} = -\frac{\sqrt{2}}{4} - \frac{\sqrt{6}}{4}$$



$$= \frac{-\sqrt{2}-\sqrt{6}}{4} = -\frac{\sqrt{2}+\sqrt{6}}{4}$$

$*$ $\cos(225^\circ - 30^\circ) = \cos(225^\circ)\cos(30^\circ) + \sin(225^\circ)\sin(30^\circ)$

$$= -\frac{\sqrt{2}}{2}\cdot\frac{\sqrt{3}}{2} - \frac{\sqrt{2}}{2}\cdot\frac{1}{2} = -\frac{\sqrt{6}}{4} - \frac{\sqrt{2}}{4} = -\frac{\sqrt{6}+\sqrt{2}}{4}$$

even/odd id

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(2) $\cos\left(-\frac{\pi}{12}\right) = \cos\left(\frac{\pi}{12}\right) = \cos(15°) = \frac{\sqrt{2}+\sqrt{6}}{4}$

(3) $\cos(173°)\cos(128°) + \sin(173°)\sin(128°)$

$= \cos(173° - 128°) = \cos(45°) = \boxed{\frac{\sqrt{2}}{2}}$.

(4) $\cos\left(\frac{\pi}{18}\right)\cos\left(\frac{\pi}{9}\right) - \sin\left(\frac{\pi}{18}\right)\sin\left(\frac{\pi}{9}\right)$

$= \cos\left(\frac{\pi}{18} + \frac{\pi}{9}\cdot\frac{2}{2}\right) = \cos\left(\frac{\pi}{18} + \frac{2\pi}{18}\right)$

$= \cos\left(\frac{3\pi}{18}\right) = \cos\left(\frac{\pi}{6}\right) = \boxed{\frac{\sqrt{3}}{2}}$

α is in Q II $\qquad$ β is in Q III

E.g. Given: $\sin(\alpha) = \frac{3}{5}$; $\cos(\beta) = -\frac{12}{13}$.

Find: (a) $\cos(\alpha - \beta)$ $\qquad$ (b) $\cos(\alpha + \beta)$.

Sol:

missing $\quad$ given $\quad$ given $\quad$ missing

(a) $\cos(\alpha - \beta) = \boxed{\cos(\alpha)}\boxed{\cos(\beta)} + \boxed{\sin(\alpha)}\boxed{\sin(\beta)}$

* Find $\cos(\alpha)$

$\sin^2(\alpha) + \cos^2(\alpha) = 1$

$\cos^2(\alpha) = 1 - \sin^2(\alpha) = 1 - \left(\frac{3}{5}\right)^2 = 1 - \frac{9}{25}$

$\cos^2(\alpha) = \frac{16}{25} \longrightarrow \cos(\alpha) = \pm\frac{4}{5}$

Since α is in QII, $\cos(\alpha) = -\frac{4}{5}$

* Find $\sin(\beta)$

$$\sin^2(\beta) + \cos^2(\beta) = 1$$

$$\sin^2(\beta) = 1 - \cos^2(\beta) = 1 - \left(-\frac{12}{13}\right)^2 = 1 - \frac{144}{169}$$

$$\sin^2(\beta) = \frac{25}{169} \rightarrow \sin(\beta) = \pm\frac{5}{13}$$

Since β is in QIII, $\sin(\beta) = -\frac{5}{13}$.

So,

$$\cos(\alpha-\beta) = \left(-\frac{4}{5}\right)\cdot\left(-\frac{12}{13}\right) + \left(\frac{3}{5}\right)\cdot\left(-\frac{5}{13}\right)$$

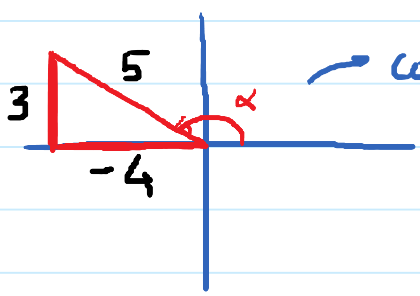
$$= \frac{48}{65} - \frac{15}{65} = \boxed{\frac{33}{65}} \rightarrow \cos(\alpha-\beta)$$

$$\cos(\alpha+\beta) = \left(-\frac{4}{5}\right)\cdot\left(-\frac{12}{13}\right) - \left(\frac{3}{5}\right)\cdot\left(-\frac{5}{13}\right)$$

$$= \frac{48}{65} + \frac{15}{65} = \boxed{\frac{63}{65}} \rightarrow \cos(\alpha+\beta)$$

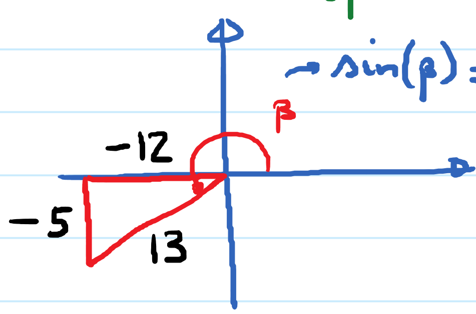
* 2^{nd} way of doing this:

* Find $\cos(\alpha)$.

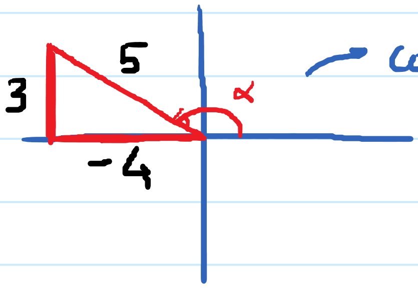


$\rightarrow \cos(\alpha) = -\frac{4}{5}$

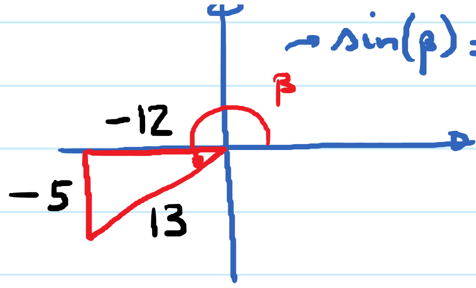
* Find $\sin(\beta)$



$\rightarrow \sin(\beta) = -\frac{5}{13}$.



$\cos(\alpha) = -\frac{4}{5}$



$\sin(\beta) = -\frac{5}{13}$.

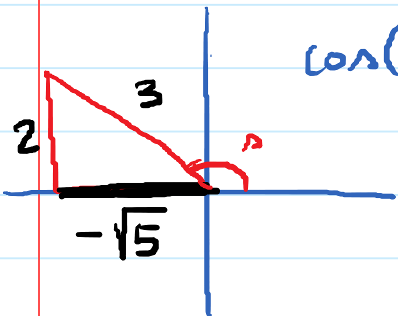
E.g. $\sin(s) = \frac{2}{3}$; $\sin(t) = -\frac{1}{3}$.

s, t are in Q II and Q IV respectively

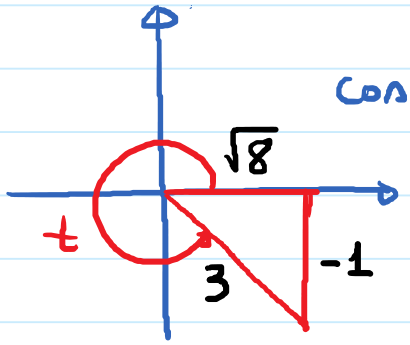
Find $\cos(s+t)$ and $\cos(s-t)$

$$\cos(s+t) = \boxed{\cos(s)}\boxed{\cos(t)} - \boxed{\sin(s)}\boxed{\sin(t)}$$

missing given



$\cos(s) = -\frac{\sqrt{5}}{3}$



$\cos(t) = \frac{\sqrt{8}}{3}$

$$= -\frac{\sqrt{5}}{3} \cdot \frac{\sqrt{8}}{3} - \frac{2}{3} \cdot \left(-\frac{1}{3}\right)$$

$\cos(s+t)$

$$= -\frac{\sqrt{40}}{9} + \frac{2}{9} = \frac{-\sqrt{40}+2}{9} = \frac{-2\sqrt{10}+2}{9}.$$

$$\cos(s-t) = -\frac{\sqrt{5}}{3} \cdot \frac{\sqrt{8}}{3} + \frac{2}{3} \cdot \left(-\frac{1}{3}\right)$$

$$= -\frac{\sqrt{40}}{9} - \frac{2}{9} = \frac{-2\sqrt{10}-2}{9}$$

$$\cos(s-t) = -\frac{2\sqrt{10}+2}{9}.$$