5.4. Sum and Difference Identities for Sine and Tangent

Thursday, March 28, 2019 8:06 AM

Recall:

$$\boxed{\begin{aligned}\cos(A+B) &= \cos(A)\cos(B) - \sin(A)\sin(B)\\ \cos(A-B) &= \cos(A)\cdot\cos(B) + \sin(A)\sin(B)\end{aligned}}$$

Sum / Difference Identities for Sine

$$\boxed{\begin{aligned}\sin(A+B) &= \sin(A)\cdot\cos(B) + \cos(A)\cdot\sin(B)\\ \sin(A-B) &= \sin(A)\cdot\cos(B) - \cos(A)\cdot\sin(B)\end{aligned}}$$

E.g. Find the exact value of $\sin(105^\circ)$

$$\begin{aligned}\sin(105^\circ) &= \sin(60^\circ + 45^\circ)\\ &= \sin(60^\circ)\cdot\cos(45^\circ) + \cos(60^\circ)\cdot\sin(45^\circ)\\ &= \frac{\sqrt{3}}{2}\cdot\frac{\sqrt{2}}{2} + \frac{1}{2}\cdot\frac{\sqrt{2}}{2}\\ &= \frac{\sqrt{6}}{4} + \frac{\sqrt{2}}{4} = \boxed{\frac{\sqrt{6}+\sqrt{2}}{4}}\end{aligned}$$

E.g. $\sin(270^\circ - \theta)$. Write this as a single function of θ.

$$\sin(270^\circ - \theta) = \sin(270^\circ)\cdot\cos\theta - \cos(270^\circ)\sin\theta$$

$$= -1\cdot\cos\theta - 0\cdot\sin\theta$$

$$= \boxed{-\cos\theta}$$

Sum / Difference Identities for Tangent.

$$\tan(A+B) = \frac{\tan(A)+\tan(B)}{1-\tan(A)\tan(B)}$$

$$\tan(A-B) = \frac{\tan(A)-\tan(B)}{1+\tan(A)\tan(B)}$$

E.g. Find the exact value of $\tan(15^\circ)$

$$\tan(60^\circ - 45^\circ) = \frac{\tan(60^\circ) - \tan(45^\circ)}{1+\tan(60^\circ)\cdot\tan(45^\circ)}$$

$$= \frac{\sqrt{3}-1}{1+\sqrt{3}\cdot 1} = \frac{\sqrt{3}-1}{1+\sqrt{3}} \cdot \frac{1-\sqrt{3}}{1-\sqrt{3}}$$

$$= \frac{\sqrt{3}-3-1+\sqrt{3}}{-2}$$

$$= \frac{2\sqrt{3}-4}{-2} = \frac{\cancel{-2}(-\sqrt{3}+2)}{\cancel{-2}}$$

$$= \boxed{-\sqrt{3}+2}$$

$$= \boxed{-\sqrt{3} + 2}$$

E.g. Simplify $\dfrac{\tan\left(\frac{3\pi}{4}\right)+\tan\left(\frac{\pi}{6}\right)}{1-\tan\left(\frac{3\pi}{4}\right)\tan\left(\frac{\pi}{6}\right)}$

$$= \tan\left(\frac{3\pi\cdot 3}{4\cdot 3}+\frac{\pi\cdot 2}{6\cdot 2}\right) = \boxed{\tan\left(\frac{11\pi}{12}\right)}$$

tangent of sum id

E.g. $\cos(s)=\dfrac{12}{13}$; $\sin(t)=-\dfrac{3}{5}$

s and t are in Q IV.

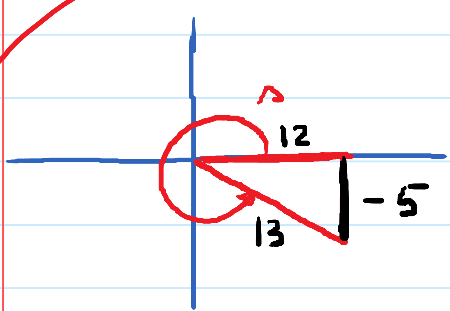
(a) Find $\sin(s+t)$ (b) $\tan(s+t)$

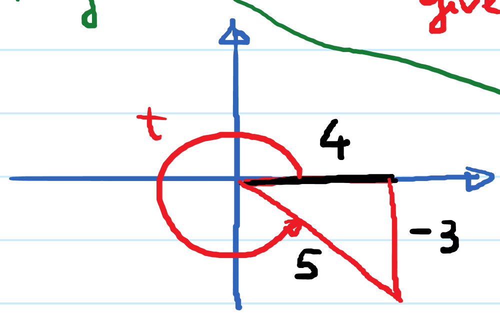
(c) Which quadrant does $s+t$ belong to ?

Sol:

(a) $\boxed{\sin(s+t)} = \boxed{\sin(s)}\,\boxed{\cos(t)} + \boxed{\cos(s)}\cdot\boxed{\sin(t)}$

missing given





$\sin(s) = -\dfrac{5}{13}$ $\cos(t) = \dfrac{4}{5}$

$$-\frac{5}{13}\cdot\frac{4}{5}+\frac{12}{13}\cdot\left(-\frac{3}{5}\right)=\frac{-20-36}{65}=\boxed{-\frac{56}{65}}$$

(b) $$\tan(s+t) = \frac{\tan(s) + \tan(t)}{1 - \tan(s)\cdot\tan(t)}$$

$$\tan(s) = \frac{\sin(s)}{\cos(s)} = \frac{-\frac{5}{13}}{\frac{12}{13}} = -\frac{5}{\cancel{13}}\cdot\frac{\cancel{13}}{12} = \boxed{-\frac{5}{12}}$$

$$\tan(t) = \frac{\sin(t)}{\cos(t)} = \frac{-\frac{3}{5}}{\frac{4}{5}} = -\frac{3}{\cancel{5}}\cdot\frac{\cancel{5}}{4} = \boxed{-\frac{3}{4}}$$

$$\tan(s+t) = \frac{-\frac{5}{12} + \left(-\frac{3}{4}\right)}{1 - \left(-\frac{5}{12}\right)\cdot\left(-\frac{3}{4}\right)}$$

$$\tan(s+t) = -\frac{56}{33}$$

(c) $\sin(\boxed{s+t}) < 0$; $\tan(\boxed{s+t}) < 0$

So, $s+t$ is in Q IV

E.g. Verify a Trig Identity

$$\boxed{\sin(2x) = 2\sin(x)\cos(x)}$$

Sol: $\text{LHS} = \sin(2x) = \sin(x+x)$

$= \sin(x)\cos(x) + \cos(x)\sin(x)$

$= 2\sin(x)\cos(x) = \text{RHS}.$