5.5. Double Angle Identities

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* Double - Angle Identities for Cosine.

I $\boxed{\cos(2A) = \cos^2(A) - \sin^2(A)}$

Verify this identity:

$$LHS = \cos(2A) = \cos(A + A)$$

$$= \cos(A)\cdot\cos(A) - \sin(A)\cdot\sin(A)$$

$$= \cos^2(A) - \sin^2(A) = RHS.$$

Immediate variations:

$$\cos(2A) = \cos^2(A) - \sin^2(A)$$

$$= \cos^2(A) - [1 - \cos^2(A)]$$

$$= 2\cos^2(A) - 1$$

II $\boxed{\cos(2A) = 2\cos^2(A) - 1}$

$$\cos(2A) = \cos^2(A) - \sin^2(A)$$

$$= 1 - \sin^2(A) - \sin^2(A)$$

$$= 1 - 2\sin^2(A).$$

III $\boxed{\cos(2A) = 1 - 2\sin^2(A)}$

Double Angle Identity for Sine.

$$\boxed{\sin(2A) = 2\sin(A)\cos(A)}$$

Verify:

$$LHS = \sin(2A) = \sin(A + A)$$

$$= \sin(A)\cos(A) + \sin(A)\cos(A)$$

$$= 2\sin(A)\cos(A) = RHS.$$

Double Angle Identity for tangent.

$$\boxed{\tan(2A) = \frac{2\tan(A)}{1 - \tan^2(A)}}$$

Verify: $LHS = \tan(2A) = \tan(A + A)$

$$= \frac{\tan(A) + \tan(A)}{1 - \tan(A)\tan(A)}$$

$$= \frac{2\tan(A)}{1 - \tan^2(A)} = RHS.$$

Summary of all Double Angle Identities.

$$\cos(2A) = \begin{cases} \cos^2(A) - \sin^2(A) \\ 2\cos^2(A) - 1 \\ 1 - 2\sin^2(A) \end{cases}$$

$$\sin(2A) = 2\sin(A)\cos(A)$$

$$\tan(2A) = \frac{2\tan(A)}{1 - \tan^2(A)}$$

E.g. Given: $\cos(\theta) = -\frac{12}{13}$ and $\sin(\theta) > 0$

Find the exact value of $\cos(2\theta)$ and $\sin(2\theta)$.

* Find $\cos(2\theta)$

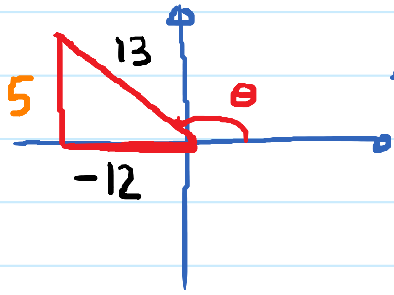
$$\cos(2\theta) = 2\cos^2(\theta) - 1$$

$$= 2\cdot\left(-\frac{12}{13}\right)^2 - 1 = \boxed{\frac{119}{169}}$$

* Find $\sin(2\theta)$

$$\sin(2\theta) = 2\underbrace{\boxed{\sin(\theta)}}_{\text{missing}}\underbrace{\boxed{\cos(\theta)}}_{\text{given}}$$

We need $\sin\theta$



$$\sin(\theta) = \frac{5}{13}.$$

So, $\sin(2\theta) = 2 \cdot \frac{5}{13} \cdot \left(-\frac{12}{13}\right) = \boxed{-\frac{120}{169}}$

Ex. Given: $\cos(2\theta) = -\frac{28}{53}$ and θ is in Q II.

Find the exact value of $\sin(\theta)$ and $\cos(\theta)$

* Find $\sin(\theta)$

$$\cos(2\theta) = 1 - 2\sin^2(\theta)$$

$$\rightarrow 2\sin^2(\theta) = 1 - \cos(2\theta)$$

$$\rightarrow \sin^2(\theta) = \frac{1-\cos(2\theta)}{2}$$

$$\rightarrow \sin(\theta) = \pm\sqrt{\frac{1-\cos(2\theta)}{2}}$$

Since θ is in Q II, $\sin(\theta) = \sqrt{\frac{1-\cos(2\theta)}{2}}$

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$$\sin(\theta) = \sqrt{\frac{1 - \left(-\frac{28}{53}\right)}{2}} \dots$$

* Find $\cos(\theta)$

$$\cos(2\theta) = 2\cos^2(\theta) - 1$$

$$\cos(2\theta) + 1 = 2\cos^2(\theta)$$

$$\cos^2(\theta) = \frac{\cos(2\theta) + 1}{2}$$

$$\cos(\theta) = \pm\sqrt{\frac{\cos(2\theta)+1}{2}}$$

Since θ is in QII, $\cos(\theta) = -\sqrt{\frac{\cos(2\theta)+1}{2}}$

$$\cos(\theta) = -\sqrt{\frac{-\frac{28}{53}+1}{2}} = \dots$$