5.6. Half-Angle Identities

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All Half-Angle Identities.

$$\boxed{\cos\left(\frac{A}{2}\right) = \pm\sqrt{\frac{1+\cos(A)}{2}} \; ; \; \sin\left(\frac{A}{2}\right) = \pm\sqrt{\frac{1-\cos(A)}{2}}}$$

The + or − depends on the quadrant that $\frac{A}{2}$ belongs to.

$$\boxed{\tan\left(\frac{A}{2}\right) = \left[\begin{array}{l} \pm\sqrt{\dfrac{1-\cos(A)}{1+\cos(A)}} \\[2ex] \dfrac{\sin(A)}{1+\cos(A)} \\[2ex] \dfrac{1-\cos(A)}{\sin(A)} \end{array}\right.}$$

E.g. Use the half angle identities to find the exact value of the given expression:

ⓐ $\sin(15°)$ ⓑ $\tan(195°)$

E.g. Given: $\cos(\theta) = -\frac{5}{8}$ and $\boxed{\frac{\pi}{2} < \theta < \pi.}$

Find $\sin\left(\frac{\theta}{2}\right)$

E.g. Given $\tan(\theta) = \frac{\sqrt{7}}{3}$ and $180° < \theta < 270°$, find $\tan\left(\frac{\theta}{2}\right)$.

(15° is in QI)

Sol

① ⓐ $\sin(\overset{\frac{A}{2}}{\boxed{15^\circ}}) = \sqrt{\dfrac{1-\cos(30^\circ)}{2}}$

$$= \sqrt{\dfrac{1-\frac{\sqrt{3}}{2}}{2}} = \sqrt{\dfrac{\frac{2-\sqrt{3}}{2}}{2}}$$

$$= \sqrt{\dfrac{2-\sqrt{3}}{4}} = \dfrac{\sqrt{2-\sqrt{3}}}{2}$$

ⓑ $\tan(\overset{A/2}{\boxed{195^\circ}}) = \dfrac{\sin(\boxed{390^\circ})}{1+\cos(\boxed{390^\circ})}$ $\rightarrow$ $360^\circ + 30^\circ$

$$= \dfrac{\sin(30^\circ)}{1+\cos(30^\circ)} = \dfrac{\frac{1}{2}}{1+\frac{\sqrt{3}}{2}}$$

$$= \dfrac{\frac{1}{2}}{\frac{2+\sqrt{3}}{2}} = \boxed{\dfrac{1}{2+\sqrt{3}}}$$

②

$$\sin\left(\frac{\theta}{2}\right) = \pm\sqrt{\dfrac{1-\cos(\theta)}{2}}$$

Which Quadrant $\frac{\theta}{2}$ is in?

Given: $\frac{\pi}{2} < \theta < \pi \rightarrow 90^\circ < \theta < 180^\circ$

$\rightarrow 45^\circ < \boxed{\frac{\theta}{2}} < 90^\circ \rightarrow$ is in QI.

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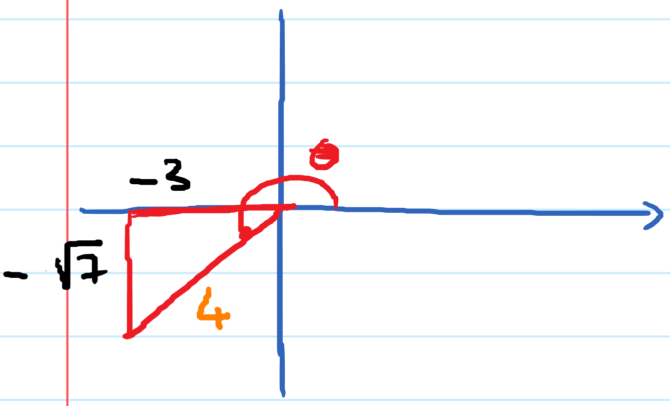
$$\sin\left(\frac{\theta}{2}\right) = \sqrt{\frac{1-\cos(\theta)}{2}} = \sqrt{\frac{1-\left(-\frac{5}{8}\right)}{2}}$$

$$= \sqrt{\frac{13}{16}} = \boxed{\frac{\sqrt{13}}{4}}$$

E.g. $\tan\left(\frac{\theta}{2}\right) = \dfrac{1-\cos(\theta)}{\sin(\theta)}$



$$\sin(\theta) = -\frac{\sqrt{7}}{4}$$

$$\cos(\theta) = \frac{-3}{4}$$