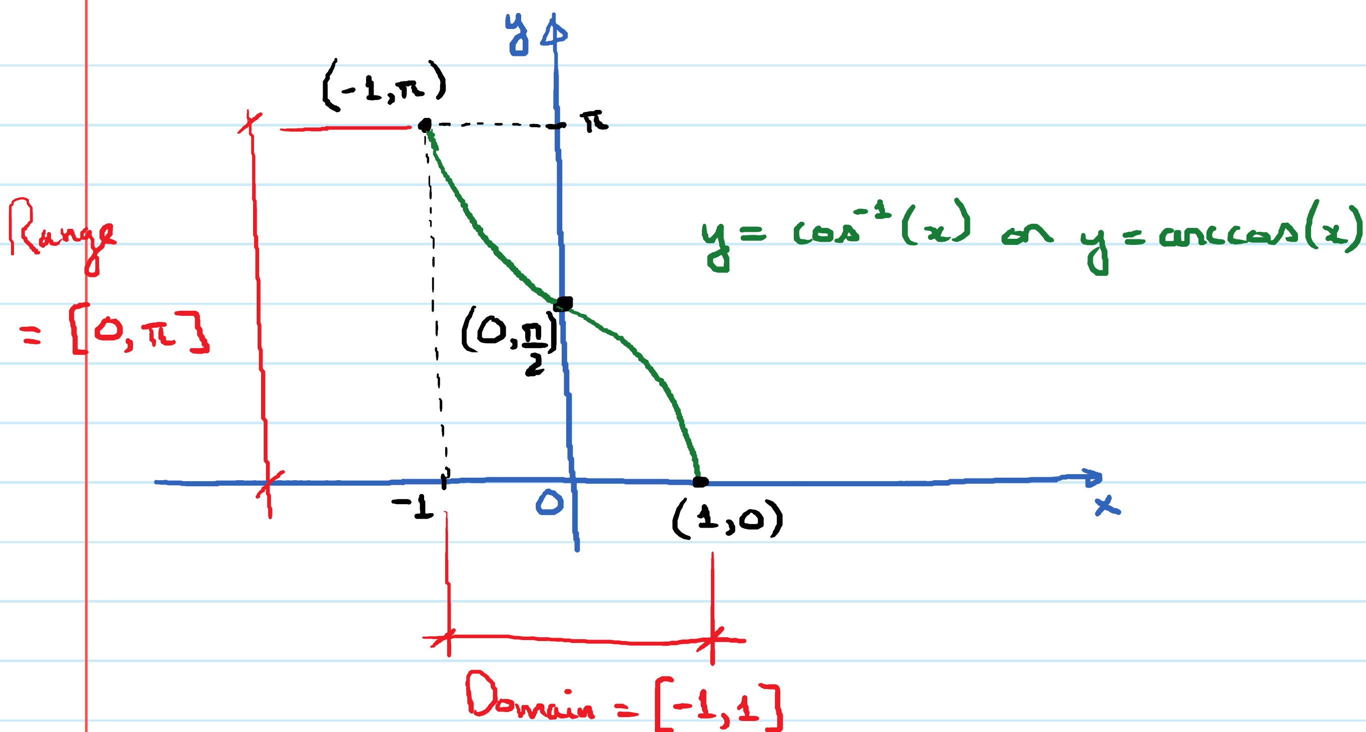


To sum up,

$\cos^{-1}(x)$ takes a # in $[-1, 1]$ and gives us the angle y in $[0, \pi]$ whose cosine equals that #.

Inverse Tangent Function:

Notation: $y = \tan^{-1}(x)$ or $y = \arctan(x)$

QI and IV

$\tan^{-1}(x)$ gives us the angle y in $\left(-\frac{\pi}{2}, \frac{\pi}{2}\right)$ such that $\tan(y) = x$

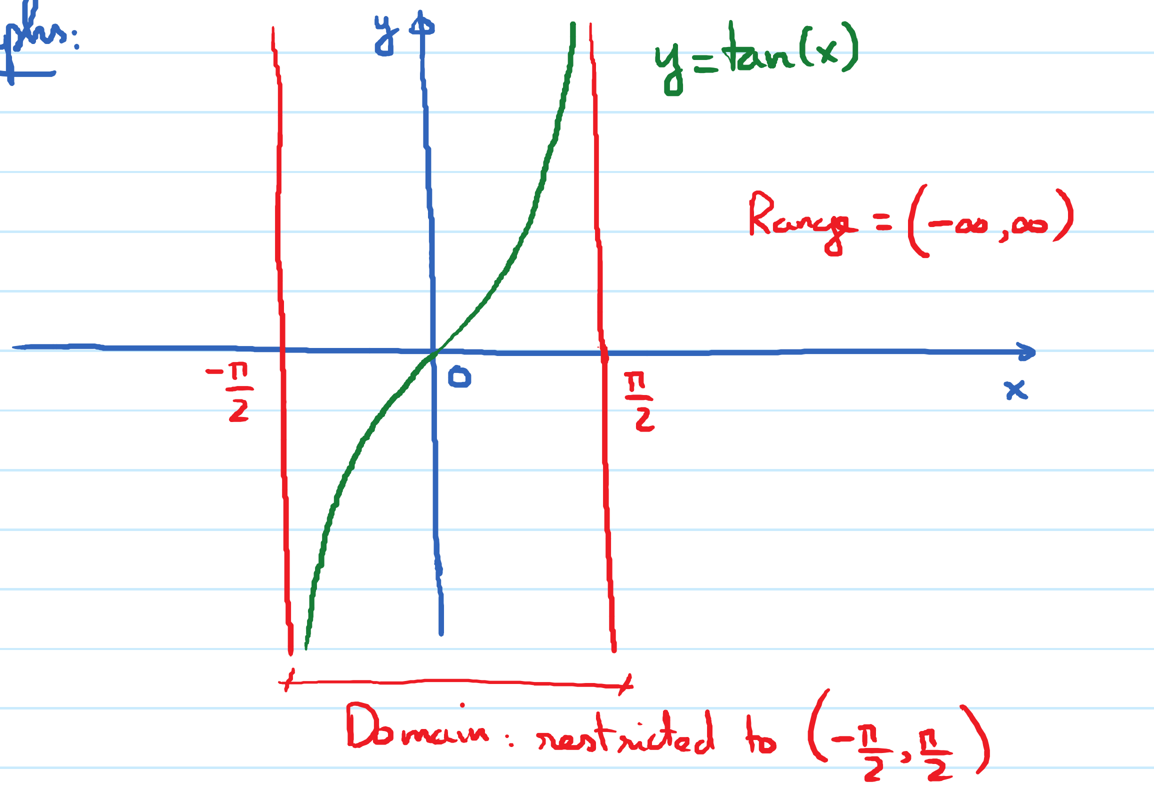
In short, $y = \tan^{-1}(x) \longleftrightarrow \tan(y) = x.$

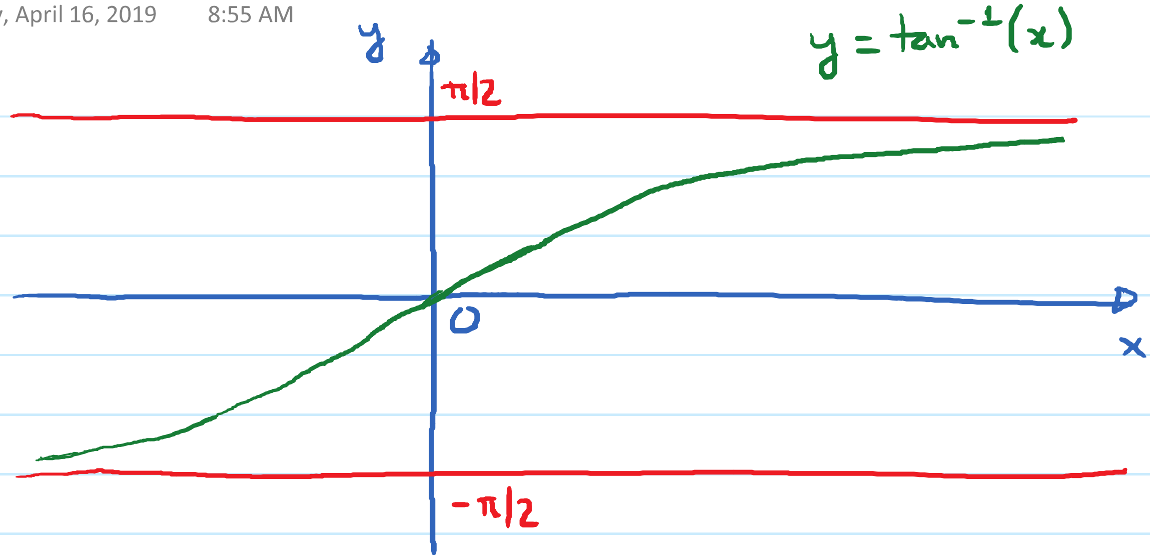
x	$y = \tan(x)$
$-\pi/2$	undefined (V.A.)
$-\pi/3$	$-\sqrt{3}$
$-\pi/4$	-1
$-\pi/6$	$-1/\sqrt{3}$
0	0
$\pi/6$	$1/\sqrt{3}$
$\pi/4$	1
$\pi/3$	$\sqrt{3}$
$\pi/2$	undefined (V.A)

x	$y = \tan^{-1}(x)$
H.A.	$-\pi/2$
$-\sqrt{3}$	$-\pi/3$
-1	$-\pi/4$
$-1/\sqrt{3}$	$-\pi/6$
0	0
$1/\sqrt{3}$	$\pi/6$
1	$\pi/4$
$\sqrt{3}$	$\pi/3$
H.A.	$\pi/2$

$$\tan^{-1}(\underbrace{-1}_{\#}) = \underbrace{-\frac{\pi}{4}}_{\text{angle in } \left(-\frac{\pi}{2}, \frac{\pi}{2}\right) \text{ whose tangent } = \#}$$

Graphs:





Domain: $= (-\infty, \infty)$. Range $= \left(-\frac{\pi}{2}, \frac{\pi}{2}\right)$

Algebraic Calculations with Inverse Trig Functions.

E.g. Find the exact value of $\tan\left(\underbrace{\arcsin\left(\frac{\sqrt{3}}{2}\right)}_{\frac{\pi}{3}}\right)$

Sol: 1st way: inside to outside

Ⓘ. $\arcsin\left(\frac{\sqrt{3}}{2}\right) = \frac{\pi}{3}$

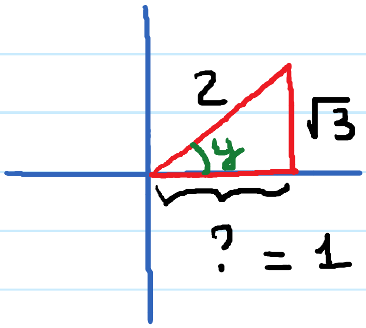
Ⓘⓘ. $\tan\left(\frac{\pi}{3}\right) = \boxed{\sqrt{3}}$ $\longrightarrow$ answer.

2nd way: $\tan\left(\underbrace{\arcsin\left(\frac{\sqrt{3}}{2}\right)}_{y}\right)$

Let $y = \arcsin\left(\frac{\sqrt{3}}{2}\right)$. Then $\sin(y) = \frac{\sqrt{3}}{2}$

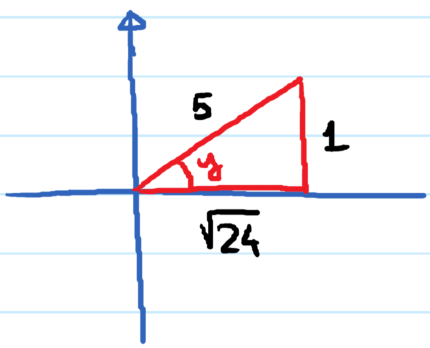
and y is in QI

Goal: Find $\tan(y)$.



$\tan(y) = \boxed{\sqrt{3}}$

E.g. Find the exact value of $\tan(\underbrace{\arcsin(\tfrac{1}{5})}_{y})$



$$\tan(y) = \frac{1}{\sqrt{24}} = \frac{\sqrt{24}}{24} = \frac{2\sqrt{6}}{24}$$

$$= \frac{\sqrt{6}}{12}.$$

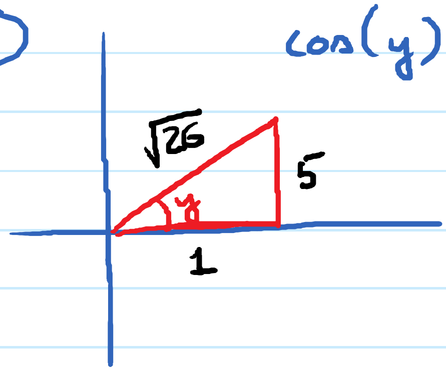
E.g. Find the exact value of

(a) $\cos(\underbrace{\tan^{-1}(5)}_{y})$

(b) $\sin(2\overbrace{\boxed{\tan^{-1}(\tfrac{3}{4})}}^{y})$

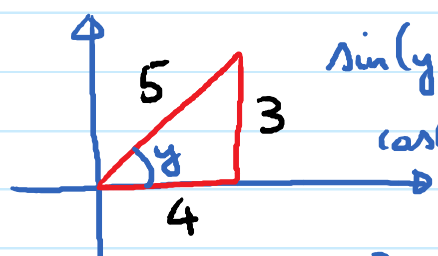
(c) $\cos(2\arctan(-\tfrac{4}{3}))$

(a)



$\cos(y) = \dfrac{1}{\sqrt{26}}$

(b) $\sin(2y) = 2\sin(y)\cos(y)$



$\sin(y) = \dfrac{3}{5}$

$\cos(y) = \dfrac{4}{5}$

$\longrightarrow \sin(2y) = 2 \cdot \dfrac{3}{5} \cdot \dfrac{4}{5} = \boxed{\dfrac{24}{25}}$