6.2. Trig Equations Part I

Thursday, April 18, 2019 8:19 AM

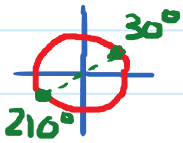
Linear Equations:

E.g. $3\tan\theta - \sqrt{3} = 0$

(a) Solve this equation for θ in $[0°, 360°)$

$\rightarrow 3\tan\theta = \sqrt{3} \rightarrow \tan\theta = \frac{\sqrt{3}}{3}$ (isolate $\tan\theta$)

$\rightarrow$ unit circle tells us: $\boxed{\theta = 30°;\ 210°}$



(b) Solve for all solutions:

$\theta = 30° + n \cdot 360°$ where $n = 0; \pm1; \pm2; \pm3; \ldots$

$\theta = 210° + n \cdot 360°$ where $n = 0; \pm1; \pm2; \pm3; \ldots$

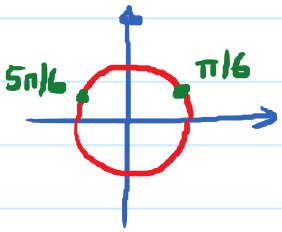
E.x. Solve the equation: $2\sin x + 3 = 4$

On $[0, 2\pi)$

$\rightarrow 2\sin x = 1$

$\rightarrow \sin x = \frac{1}{2}$

$\rightarrow x = \frac{\pi}{6}, \frac{5\pi}{6}$



All solutions

$x = \frac{\pi}{6} + n \cdot 2\pi$

$x = \frac{5\pi}{6} + n \cdot 2\pi$

$n = 0; \pm1, \pm2 \ldots$

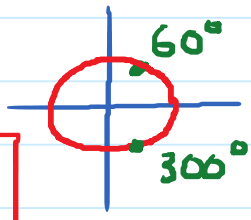
Quadratic Equations solved by Root Extraction.

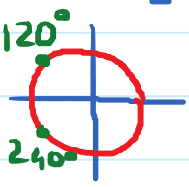
E.g. Solve $4\cos^2 x - 1 = 0$

(a) On $[0°, 360°)$

$\rightarrow 4\cos^2 x = 1 \rightarrow \cos^2 x = \frac{1}{4} \rightarrow \cos x = \pm\sqrt{\frac{1}{4}} = \pm\frac{1}{2}$

$\cos x = \frac{1}{2}$ or $\cos x = -\frac{1}{2}$





$x = \boxed{60°, 300°}$ $x = \boxed{120°, 240°}$

(b) All solutions:

$x = 60° + n\cdot 360°;\ 300° + n\cdot 360°;\ 120° + n\cdot 360°;$

$240° + n\cdot 360°$; where $n = 0, \pm 1, \pm 2, \ldots$

Ex: Solve $\tan^2 x - 3 = 0$

(a) On $[0, 2\pi)$

$\rightarrow \tan^2 x = 3$

$\rightarrow \tan x = \pm\sqrt{3}$

$x = \underbrace{\frac{\pi}{3}, \frac{4\pi}{3}}_{\tan x = \sqrt{3}};\ \underbrace{\frac{2\pi}{3}, \frac{5\pi}{3}}_{\tan x = -\sqrt{3}}$

(b) All solutions

Add $n\cdot 2\pi$ to each of the solutions in (a).

Quadratic by Factoring

E.g. $-2\sin^2 x = 3\sin x + 1$.

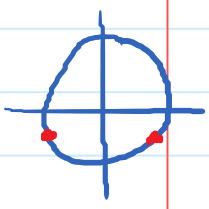
(a) Solve on $[0, 2\pi)$

$$0 = 2\sin^2 x + 3\sin x + 1$$

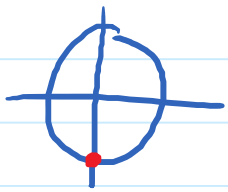
$$0 = (2\sin x + 1)(\sin x + 1)$$

$$\Rightarrow 2\sin x + 1 = 0 \quad \text{or} \quad \sin x + 1 = 0$$

$$\sin x = -\frac{1}{2} \quad \text{or} \quad \sin x = -1$$



$$x = \frac{7\pi}{6}, \frac{11\pi}{6} \qquad x = \frac{3\pi}{2}$$



(b) All solutions:

Add $n \cdot 2\pi$ to each of the above.

E.x. Solve the given equation on $[0°, 360°)$.

(a) $\tan^2 x + \tan x - 2 = 0$

(b) $\sec^2 x \tan x = 2\tan x$.

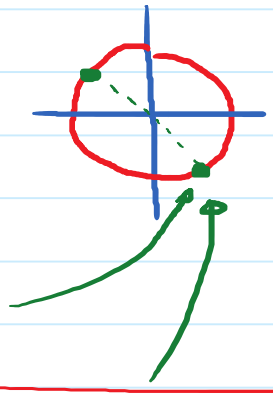
(a) $\tan^2 x + \tan x - 2 = 0$

$(\tan x + 2)(\tan x - 1) = 0$

$\rightarrow \tan x = 1$ or $\tan x = -2$

$x = \boxed{45^\circ, 225^\circ}$

$x = -63.435^\circ$



Add $360^\circ \rightarrow x = 296.565^\circ$

Subtract $180^\circ \rightarrow x = 116.565^\circ$

(b) $\sec^2 x \tan x = 2 \tan x$.

$\sec^2 x \tan x - 2 \tan x = 0$

$\tan x (\sec^2 x - 2) = 0$

$\tan x = 0$ or $\sec^2 x - 2 = 0$

$x = \boxed{0; 180^\circ}$

$\sec^2 x = 2 \rightarrow \sec x = \pm\sqrt{2}$

$\sec x = \sqrt{2}$ or $\sec x = -\sqrt{2}$

$\cos x = \frac{1}{\sqrt{2}}$ or $\cos x = -\frac{1}{\sqrt{2}}$

$x = \boxed{45^\circ, 315^\circ}$ $x = \boxed{135^\circ, 225^\circ}$

Use Identities and Factoring.

E.g. Solve on $[0°, 360°)$

(a) $2\sin\theta - 1 = \csc\theta$; (b) $5 + 5\tan^2\theta = 6\sec\theta$

Sol: (a) $2\sin\theta - 1 = \frac{1}{\sin\theta}$

$(2\sin\theta - 1)\sin\theta = 1$

$2\sin^2\theta - \sin\theta - 1 = 0$

$(2\sin\theta + 1)(\sin\theta - 1) = 0$

$\sin\theta = -\frac{1}{2}$; $\sin\theta = 1$

$\theta = 210°, 330°$; $90°$

(b) $5 + 5\tan^2\theta = 6\sec\theta$

$5(1 + \tan^2\theta) = 6\sec\theta$

$5\sec^2\theta = 6\sec\theta$

$5\sec^2\theta - 6\sec\theta = 0$; $\sec\theta(5\sec\theta - 6) = 0$

$\sec\theta = 0$ or $\sec\theta = \frac{6}{5} \rightarrow \cos\theta = \frac{5}{6}$

$\frac{1}{\cos\theta} = 0$ No sol

$\rightarrow \cos^{-1}\left(\frac{5}{6}\right)$

Square both sides and use identities:

E.g. $\tan x + \sqrt{3} = \sec x$. Solve for x in $[0, 2\pi)$

→ Square both sides:

$$(\tan x + \sqrt{3})^2 = \sec^2 x$$

$$(\tan x + \sqrt{3})(\tan x + \sqrt{3}) = \sec^2 x$$

$$\tan^2 x + 2\sqrt{3}\tan x + 3 = \sec^2 x$$

↓ Pythagorean Id

$$\tan^2 x + 2\sqrt{3}\tan x + 3 = 1 + \tan^2 x$$

$$2\sqrt{3}\tan x = -2$$

$$\tan x = -\frac{2}{2\sqrt{3}}$$

$$\tan x = -\frac{1}{\sqrt{3}}$$

→ $x = \frac{5\pi}{6}$; $\frac{11\pi}{6}$

Check answers:

$x = \frac{5\pi}{6}$: $\tan x + \sqrt{3} = \sec x$

$-\frac{1}{\sqrt{3}} + \sqrt{3} = \frac{2}{\sqrt{3}}$ ← $\frac{2}{\sqrt{3}}$

only solution.

$x = \frac{11\pi}{6}$

$\tan x + \sqrt{3} = \sec x$

$-\frac{1}{\sqrt{3}} + \sqrt{3} = \frac{2}{\sqrt{3}}$ ✓