6.3 – Trig Equations Part 2.

Equations with trig functions of multiples of angles

E.g. Solve for θ in $[0°, 360°)$:

$$\cos(\boxed{2\theta}) = \frac{\sqrt{3}}{2} \longrightarrow \text{angle}$$

$$0° \leq \theta < 360° \longrightarrow 0° \leq 2\theta < 720°$$

→ Solve the given equation for the angle (2θ) in $[0°, 720°)$. Once you get 2θ, you solve for θ by itself.

$$\cos(\boxed{2\theta}) = \frac{\sqrt{3}}{2} \qquad \text{in } [0°, 720°)$$

$$\longrightarrow 2\theta = \boxed{30°, 330°, 390°, 690°}$$

$$\longrightarrow \boxed{\theta = 15°, 165°, 195°, 345°} \longrightarrow \text{Solution.}$$

Ex.1. Solve for x in $[0°, 360°)$: $3\tan(3x) = \sqrt{3}$

Ex.2 Solve for x in $[0, 2\pi)$: $\sqrt{2}\sin(3x) - 1 = 0$

Sol:

① $\qquad 0° \leq x < 360° \longrightarrow 0° \leq 3x < 1080°$

$$\tan(\boxed{3x}) = \frac{\sqrt{3}}{3} \longrightarrow 3x = 30°, 210°, 390°, 570°, 750°, 930°$$

$\longrightarrow x = 10°, 70°, 130°, 190°, 250°, 310°.$

② $0 \leq x < 2\pi \longrightarrow 0 \leq 3x < 6\pi$

$\sin(3x) = \dfrac{1}{\sqrt{2}} \longrightarrow 3x = \dfrac{\pi}{4}, \dfrac{3\pi}{4}, \dfrac{9\pi}{4}, \dfrac{11\pi}{4}, \dfrac{17\pi}{4},$

$$\dfrac{19\pi}{4}$$

$\longrightarrow x = \dfrac{\pi}{12}, \dfrac{3\pi}{12}, \dfrac{9\pi}{12}, \dfrac{11\pi}{12}, \dfrac{17\pi}{12}, \dfrac{19\pi}{12}$

$$\dfrac{\pi}{4} \qquad \dfrac{3\pi}{4}$$

Ex : 1. $\cos\left(\dfrac{x}{2}\right) = \sqrt{2} - \cos\left(\dfrac{x}{2}\right).$ Solve for x in $[0°, 360°)$

   2. $2\sqrt{3} \sin\left(\dfrac{x}{2}\right) = 3.$ Solve for x in $[0, 2\pi)$

Solved in class