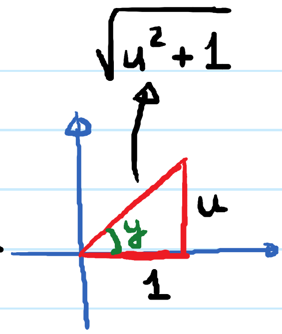
Final Exam Review

Tuesday, April 30, 2019 8:58 AM

MC

(#1) $\cos(\boxed{\arctan u})$ → y

Let $y = \arctan u \longrightarrow \tan y = u = \frac{u}{1}$



Want: $\cos y$.

$$\cos y = \frac{1}{\sqrt{u^2+1}} \cdot \frac{\sqrt{u^2+1}}{\sqrt{u^2+1}} = \frac{\sqrt{u^2+1}}{u^2+1}$$

Ans: $$\boxed{\cos(\arctan u) = \frac{\sqrt{u^2+1}}{u^2+1}}$$

(#2)

$$\theta = \sin^{-1}\left(\sqrt{\frac{(40)^2}{2\cdot(40)^2 + 64\cdot(5.7)}}\right) = \boxed{42^\circ}$$

(#3) $2\sin^2 x + \sin x = 1$

$\longrightarrow 2\sin^2 x + \sin x - 1 = 0$

$(2\sin x - 1)(\sin x + 1) = 0$

$2\sin x - 1 = 0 \quad ; \quad \sin x + 1 = 0$

$\sin x = \frac{1}{2}$ or $\sin x = -1$

$x = \frac{\pi}{6}, \frac{5\pi}{6}$ or $x = \frac{3\pi}{2}$.

All solutions:

$$\boxed{x = \frac{\pi}{6} + 2n\pi \,;\, \frac{5\pi}{6} + 2n\pi,\, \frac{3\pi}{2} + 2n\pi.}$$

#4

$$\sin^2 x + \sin x = 0$$

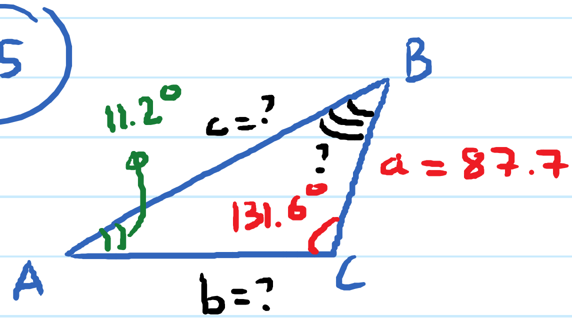
$$\sin x(\sin x + 1) = 0$$

$\sin x = 0$ or $\sin x = -1$

$x = 0, \pi$ $\qquad$ $x = \frac{3\pi}{2}$

All solutions: $\boxed{n\pi \,;\, \frac{3\pi}{2} + 2n\pi}$

#5



$$\angle B = 180° - (11.2° + 131.6°) = \boxed{37.2°}$$

$$\frac{a}{\sin(A)} = \frac{b}{\sin(B)} \longrightarrow b = \frac{a \cdot \sin(B)}{\sin(A)}$$

$$b = \frac{(87.7) \cdot \sin(37.2°)}{\sin(11.2°)} \simeq \boxed{273}$$

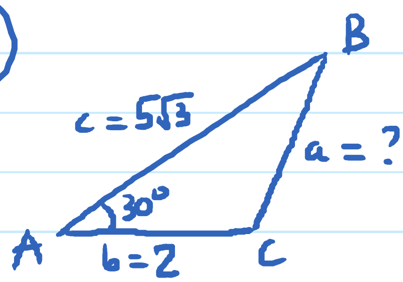
$$\frac{a}{\sin(A)} = \frac{c}{\sin(C)} \rightarrow c = \frac{a\sin(C)}{\sin(A)}$$

$$c = \frac{87.7 \cdot \sin(131.6^\circ)}{\sin(11.2^\circ)} = \boxed{337.6.}$$

(6) Area $= \frac{1}{2} bc \sin A = \frac{1}{2} \cdot (10.3) \cdot (6.8) \sin(28.9^\circ)$

$= \boxed{16.9}$

#7



Law of cosines:

$$a^2 = b^2 + c^2 - 2bc \cos A.$$

$$a^2 = (2)^2 + (5\sqrt{3})^2 - 2 \cdot 2 \cdot 5\sqrt{3} \cdot \cos 30^\circ$$

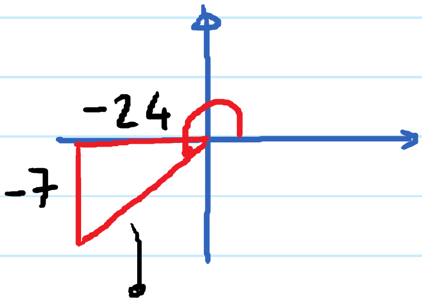
$$a = \sqrt{(2)^2 + (5\sqrt{3})^2 - 2 \cdot 2 \cdot 5\sqrt{3} \cdot \cos(30^\circ)} = \boxed{7}$$

#8 $\sin(\underbrace{\boxed{8x}}^{\theta}) \cos(\underbrace{\boxed{8x}}^{\theta}) = \boxed{\frac{1}{2}\sin(16x)}$

Double angle Identity for sine:

$$\boxed{\begin{aligned} \sin(2\theta) &= 2\sin\theta\cos\theta \\ \frac{1}{2}\sin(2\theta) &= \sin\theta\cos\theta \end{aligned}}$$

#9 $\cos(2\theta) = 2\cos^2\theta - 1$



$\cos\theta = -\dfrac{24}{25}$

$\sqrt{(-7)^2 + (-24)^2} = 25$

$\cos(2\theta) = 2\cdot\left(-\dfrac{24}{25}\right)^2 - 1 = \boxed{\dfrac{527}{625}}$

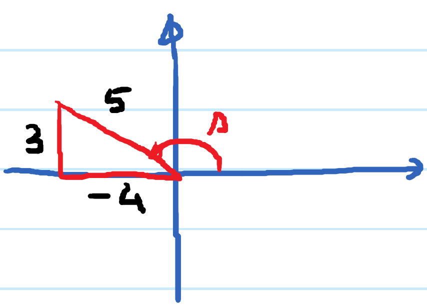
sine of difference

#10 $\sin\left(\dfrac{\pi}{4} - x\right) = \sin\dfrac{\pi}{4}\cos x - \cos\dfrac{\pi}{4}\sin x$

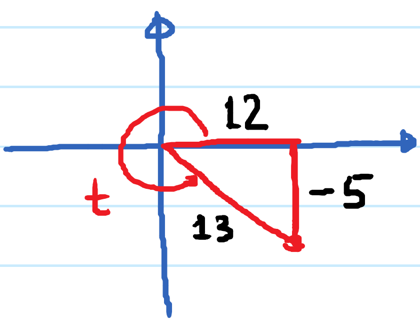
$= \boxed{\dfrac{\sqrt{2}}{2}\cos x - \dfrac{\sqrt{2}}{2}\sin x}$

cosine of difference

#11 $\cos(s-t) = \underbrace{\boxed{\cos(s)\cos(t)}}_{\text{given}} + \underbrace{\boxed{\sin(s)\sin(t)}}_{\text{missing}}$



$\sin(s) = \dfrac{3}{5}$



$\sin(t) = -\dfrac{5}{13}$

$$\cos(s-t) = \left(-\frac{4}{5}\right)\cdot\left(\frac{12}{13}\right) + \frac{3}{5}\cdot\left(-\frac{5}{13}\right)$$

$$= \boxed{-\frac{63}{65}}$$

#12 $$\frac{\sin x \cos x}{\tan x} = \frac{\sin x \cos x}{\frac{\sin x}{\cos x}}$$

quotient identity

$$= \cancel{\sin x} \cos x \cdot \frac{\cos x}{\cancel{\sin x}} = \boxed{\cos^2 x}$$

S.A.

#13 $$\cos(-x)\cos x - \sin(-x)\sin x$$

$$= \cos x \cos x + \sin x \sin x$$

even/odd

$$= \cos^2 x + \sin^2 x = 1$$

#14 $$\cos\Big(2\underbrace{\boxed{\arcsin \tfrac{1}{4}}}_{y}\Big) = \cos(2y) = \underbrace{1 - 2\sin^2 y}$$

Let $y = \arcsin \frac{1}{4} \rightarrow \sin y = \frac{1}{4}$

$$1 - 2\left(\frac{1}{4}\right)^2 = \boxed{\frac{7}{8}}$$

#15 $R = \frac{1}{32} v^2 \sin 2\theta$

Given $R = 3000$; $v = 2000$

$$3000 = \frac{1}{32} \cdot (2000)^2 \sin(2\theta)$$

$$\sin(\boxed{2\theta}) = \frac{3000}{\frac{1}{32}(2000)^2} = 0.024$$
?

$$\sin(\boxed{2\theta}) = 0.024$$
?

$$2\theta = \sin^{-1}(0.024)$$

$$\theta = \frac{\sin^{-1}(0.024)}{2} = \boxed{0.69^\circ}$$

#16

$$\frac{\sin 22^\circ}{1 + \cos 22^\circ} = \tan(11^\circ)$$

Half angle for tangent: $\tan\left(\frac{\theta}{2}\right) = \frac{\sin\theta}{1 + \cos\theta}$

#17 $\cos 2\theta = -\frac{3}{8}$; $\frac{\pi}{2} < \theta < \pi$.

Find $\sin\theta$.

$$\cos(2\theta) = 1 - 2\sin^2\theta$$

$$-\frac{3}{8} = 1 - 2\sin^2\theta$$

$$-\frac{3}{8} - 1 = -2\sin^2\theta \longrightarrow -\frac{11}{8} = -2\sin^2\theta$$

$$\sin^2\theta = \frac{-\frac{11}{8}}{-2} = \frac{11}{16}$$

$$\sin\theta = \pm\sqrt{\frac{11}{16}} = \pm\frac{\sqrt{11}}{4}$$

θ is in Q II, so, $\sin\theta = \boxed{\frac{\sqrt{11}}{4}}$.

18 $$\frac{\sin\theta}{1+\sin\theta} - \frac{\sin\theta}{1-\sin\theta}$$

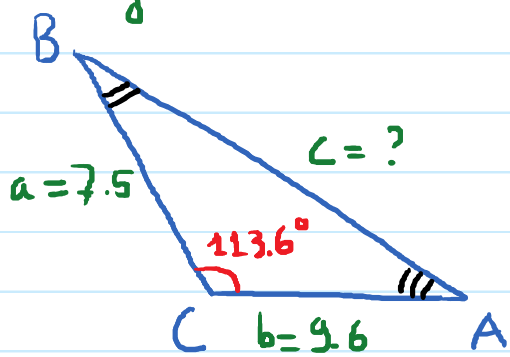
$$\frac{\sin\theta}{1+\sin\theta}\cdot\frac{1-\sin\theta}{1-\sin\theta} - \frac{\sin\theta}{1-\sin\theta}\cdot\frac{1+\sin\theta}{1+\sin\theta}$$

$$= \frac{\sin\theta\cdot(1-\sin\theta) - \sin\theta\,(1+\sin\theta)}{(1+\sin\theta)(1-\sin\theta)}$$

$$= \frac{\cancel{\sin\theta} - \sin^2\theta - \cancel{\sin\theta} - \sin^2\theta}{1-\sin^2\theta}$$

$$= \frac{-2\sin^2\theta}{\cos^2\theta} = \boxed{-2\tan^2\theta}$$

Essay.



$$c^2 = a^2 + b^2 - 2ab\cos C$$

$$c = \sqrt{(7.5)^2 + (9.6)^2 - 2(7.5)(9.6)\cos(113.6^\circ)}$$

$$c = \boxed{14.4}$$

$$\cos(A) = \frac{b^2 + c^2 - a^2}{2bc} = \frac{(9.6)^2 + (14.4)^2 - (7.5)^2}{2\cdot(9.6)\cdot(14.4)} = \boxed{\ldots\ldots}$$

$$\angle A = \cos^{-1}\left(\boxed{\ldots\ldots}\right) = \boxed{28.4^\circ}$$

$$\angle B = 180^\circ - (113.6^\circ + 28.4^\circ) = \boxed{38^\circ}$$

20) $\sqrt{3}\sec(2\theta) = 2$ over $[0^\circ, 360^\circ)$

$$\sec(2\theta) = \frac{2}{\sqrt{3}} \qquad 0^\circ \leq \theta < 360^\circ$$

$$\boxed{0^\circ \leq 2\theta < 720^\circ}$$

$$\cos(2\theta) = \frac{\sqrt{3}}{2}$$

$$2\theta = 30^\circ, 330^\circ, 390^\circ, 690^\circ$$

$$\boxed{\theta = 15^\circ, 165^\circ, 195^\circ, 345^\circ}$$