Test 3 Review

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(1) Pythagorean Identity: $\cot^2(\theta) + 1 = \csc^2(\theta)$

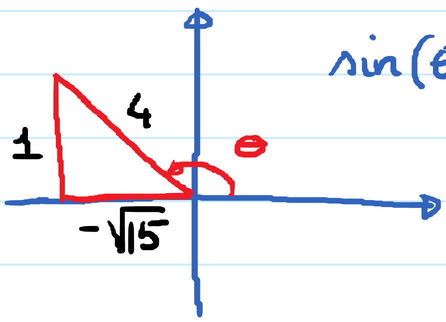
$$(-\sqrt{15})^2 + 1 = \csc^2(\theta)$$

$$\longrightarrow \csc^2(\theta) = 16 \rightarrow \csc(\theta) = \pm 4$$

Since θ is QII, $\csc(\theta) = \dfrac{1}{\sin(\theta)} > 0.$

Answer: $\csc(\theta) = 4.$

2nd method:



$\sin(\theta) = \dfrac{1}{4}$; $\csc(\theta) = \dfrac{1}{\sin(\theta)} = 4.$

(2) Pythagorean Identity:

$$\boxed{\sec^2 x} - 1 = \tan^2 x$$

(3)

$$\frac{\sin x}{\tan x} = \frac{\sin x}{\frac{\sin x}{\cos x}} = \frac{\cancel{\sin x}}{1} \cdot \frac{\cos x}{\cancel{\sin x}} = \boxed{\cos x}$$

quotient identity

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(4) $$\frac{\sin\theta}{\cos\theta} + \frac{\cos\theta}{\sin\theta} = ?$$

$$\frac{\sin\theta}{\cos\theta}\cdot\frac{\sin\theta}{\sin\theta} + \frac{\cos\theta}{\sin\theta}\cdot\frac{\cos\theta}{\cos\theta}$$

$$= \frac{\sin^2\theta + \cos^2\theta}{\sin\theta\cdot\cos\theta} = \frac{1}{\sin\theta\cdot\cos\theta}$$

$$= \frac{1}{\sin\theta}\cdot\frac{1}{\cos\theta} = \csc\theta\cdot\sec\theta$$

(5) $$\cos\left(\theta+\frac{\pi}{2}\right) = \cos\theta\cdot\cancelto{0}{\cos\frac{\pi}{2}} - \sin\theta\cdot\cancelto{1}{\sin\frac{\pi}{2}}$$

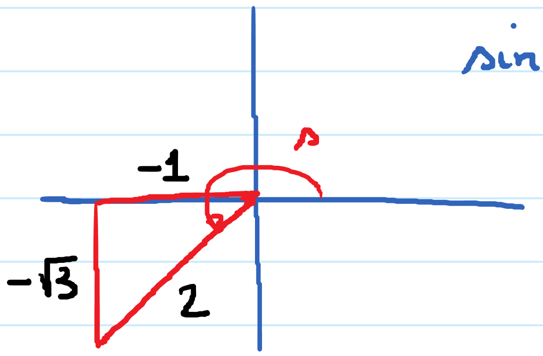
$$= \boxed{-\sin\theta}$$

Cosine of a sum

cosine of a difference

(6) $$\cos(s-t) = \underbrace{\boxed{\cos(s)}\boxed{\cos(t)}}_{\text{given}} + \underset{\text{missing}}{\boxed{\sin(s)\sin(t)}}$$

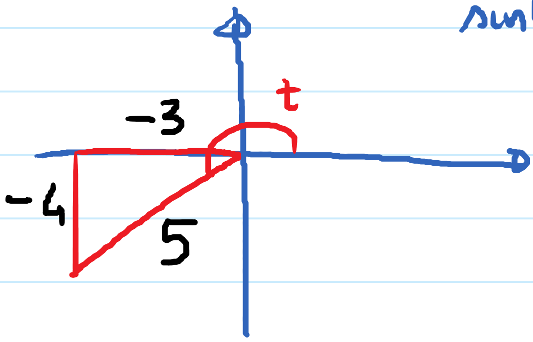
* Find $\sin(s)$



$$\sin(s) = -\frac{\sqrt{3}}{2}$$

* Find $\sin(t)$

$$\sin(t) = -\frac{4}{5}$$



So, $$\cos(s-t) = -\frac{1}{2}\cdot\left(-\frac{3}{5}\right) + \left(-\frac{\sqrt{3}}{2}\right)\cdot\left(-\frac{4}{5}\right)$$

$$= \frac{3}{10} + \frac{4\sqrt{3}}{10} = \frac{3+4\sqrt{3}}{10}$$

(7) Done on the board.

(8) $\sin\left(\frac{\pi}{4} - x\right) = \sin\left(\frac{\pi}{4}\right)\cos(x) - \sin(x)\cos\left(\frac{\pi}{4}\right)$

$= \boxed{\frac{\sqrt{2}}{2}\cos(x) - \frac{\sqrt{2}}{2}\sin(x)}$

(9) $\cos(2\theta) = 1 - 2\sin^2(\theta)$

$= 1 - 2\cdot\left(\frac{12}{13}\right)^2 = \boxed{-\frac{119}{169}}$

(10) $4\sin(2x)\cos(2x) = 2\cdot 2\sin(\boxed{2x})\cos(\boxed{2x})$

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$= \boxed{2\sin(4x)}$

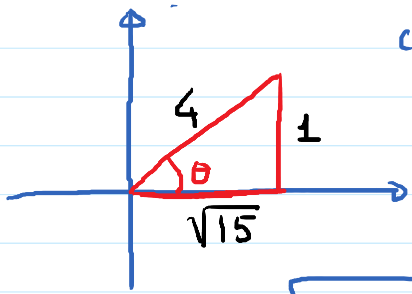
(11) $\cos\left(\frac{\theta}{2}\right) = \pm\sqrt{\frac{1+\cos(\theta)}{2}}$

Half angle identity

$0^\circ < \theta < 90^\circ \rightarrow 0^\circ < \frac{\theta}{2} < 45^\circ$

$\rightarrow \frac{\theta}{2}$ is in QI.

$\cos\left(\frac{\theta}{2}\right) = \sqrt{\frac{1+\cos(\theta)}{2}}$



$$\cos(\theta) = \frac{\sqrt{15}}{4}$$

$$\cos\left(\frac{\theta}{2}\right) = \sqrt{\frac{\frac{1\cdot 4}{1\cdot 4} + \frac{\sqrt{15}}{4}}{2}} = \sqrt{\frac{\frac{4+\sqrt{15}}{4}}{2}}$$

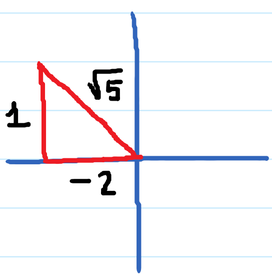
$$= \sqrt{\frac{4+\sqrt{15}}{8}} = \frac{\sqrt{4+\sqrt{15}}}{\sqrt{8}} = \frac{\sqrt{4+\sqrt{15}}}{2\sqrt{2}} \cdot \frac{\sqrt{2}}{\sqrt{2}}$$

$$= \frac{\sqrt{8+2\sqrt{15}}}{4}$$

(12) Half-angle identity for tangent

$$\frac{\sin 52^\circ}{1+\cos 52^\circ} = \tan(26^\circ)$$

(13)



$$\sin\theta = \frac{1}{\sqrt{5}} = \frac{\sqrt{5}}{5}.$$

(14)

$$\cos\frac{7\pi}{12}\cos\frac{5\pi}{12} + \sin\frac{7\pi}{12}\sin\frac{5\pi}{12}$$

$$= \cos\left(\frac{7\pi}{12} - \frac{5\pi}{12}\right) = \cos\left(\frac{2\pi}{12}\right) = \cos\left(\frac{\pi}{6}\right)$$

$$= \boxed{\frac{\sqrt{3}}{2}}$$

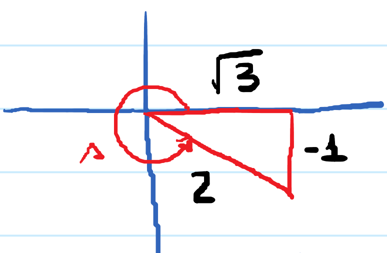
(15) $$\tan\left(x - \frac{\pi}{3}\right) = \frac{\tan x - \tan\frac{\pi}{3}}{1 + \tan x \tan\frac{\pi}{3}}$$

$$= \boxed{\frac{\tan x - \sqrt{3}}{1 + \sqrt{3}\tan x}}$$

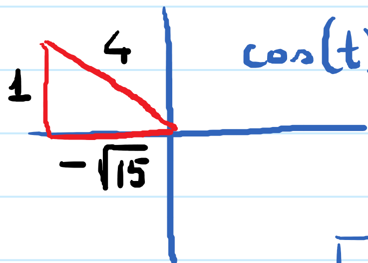
(16) $$\sin(s-t) = \boxed{\sin(s)}\,\cos(t) - \cos(s)\,\boxed{\sin(t)}$$

Given: $\sin(s) = -\frac{1}{2}$ and s is in Q IV

$\sin(t) = \frac{1}{4}$ and t is in Q II.



$\cos(s) = \frac{\sqrt{3}}{2}$



$\cos(t) = -\frac{\sqrt{15}}{4}$

$$\sin(s-t) = -\frac{1}{2}\cdot\left(-\frac{\sqrt{15}}{4}\right) - \frac{\sqrt{3}}{2}\cdot\frac{1}{4} = \boxed{\frac{\sqrt{15}-\sqrt{3}}{8}}$$

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Double angle identity for cosine

17) $\cos^2(4x) - \sin^2(4x) = \cos(8x)$

18) $\sqrt{\dfrac{1+\cos 26^\circ}{2}} = \cos(13^\circ)$

Essay:

19) $\dfrac{1-\sec\theta}{\tan\theta} + \dfrac{\tan\theta}{1-\sec\theta} = -2\csc\theta$

$$LHS = \frac{1-\sec\theta}{\tan\theta}\cdot\frac{1-\sec\theta}{1-\sec\theta} + \frac{\tan\theta}{1-\sec\theta}\cdot\frac{\tan\theta}{\tan\theta}$$

$$= \frac{(1-\sec\theta)^2 + \tan^2\theta}{\tan\theta\cdot(1-\sec\theta)}$$

$$= \frac{(1-\sec\theta)(1-\sec\theta) + \tan^2\theta}{\tan\theta\cdot(1-\sec\theta)}$$

$$= \frac{1 - 2\sec\theta + \sec^2\theta + \tan^2\theta}{\tan\theta\cdot(1-\sec\theta)}$$

$$= \frac{\sec^2\theta - 2\sec\theta + \sec^2\theta}{\tan\theta\cdot(1-\sec\theta)}$$

$$= \frac{-2\sec\theta + 2\sec^2\theta}{\tan\theta(1-\sec\theta)} = \frac{-2\sec\theta\,\cancel{(1-\sec\theta)}}{\tan\theta\,\cancel{(1-\sec\theta)}}$$

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$$= -\frac{2\sec\theta}{\tan\theta} = -\frac{2\cdot\frac{1}{\cos\theta}}{\frac{\sin\theta}{\cos\theta}} = -\frac{2}{\cancel{\cos\theta}}\cdot\frac{\cancel{\cos\theta}}{\sin\theta}$$

$$= -\frac{2}{\sin\theta} = -2\csc\theta = RHS.$$

20 $\cos(4\theta) = \cos^4\theta - 6\sin^2\theta\cos^2\theta + \sin^4\theta$

$$LHS = \cos(4\theta) = \cos^2(2\theta) - \sin^2(2\theta)$$

$$= \left[\underbrace{\cos(2\theta)}\right]^2 - \left[\sin(2\theta)\right]^2$$

$$= \left[\cos^2\theta - \sin^2\theta\right]^2 - \left[2\sin\theta\cos\theta\right]^2$$

$$= \cos^4\theta - 2\cos^2\theta\sin^2\theta + \sin^4\theta - 4\sin^2\theta\cos^2\theta$$

$$= \cos^4\theta - 6\sin^2\theta\cos^2\theta + \sin^4\theta = RHS.$$