2.1. A Preview of Calculus

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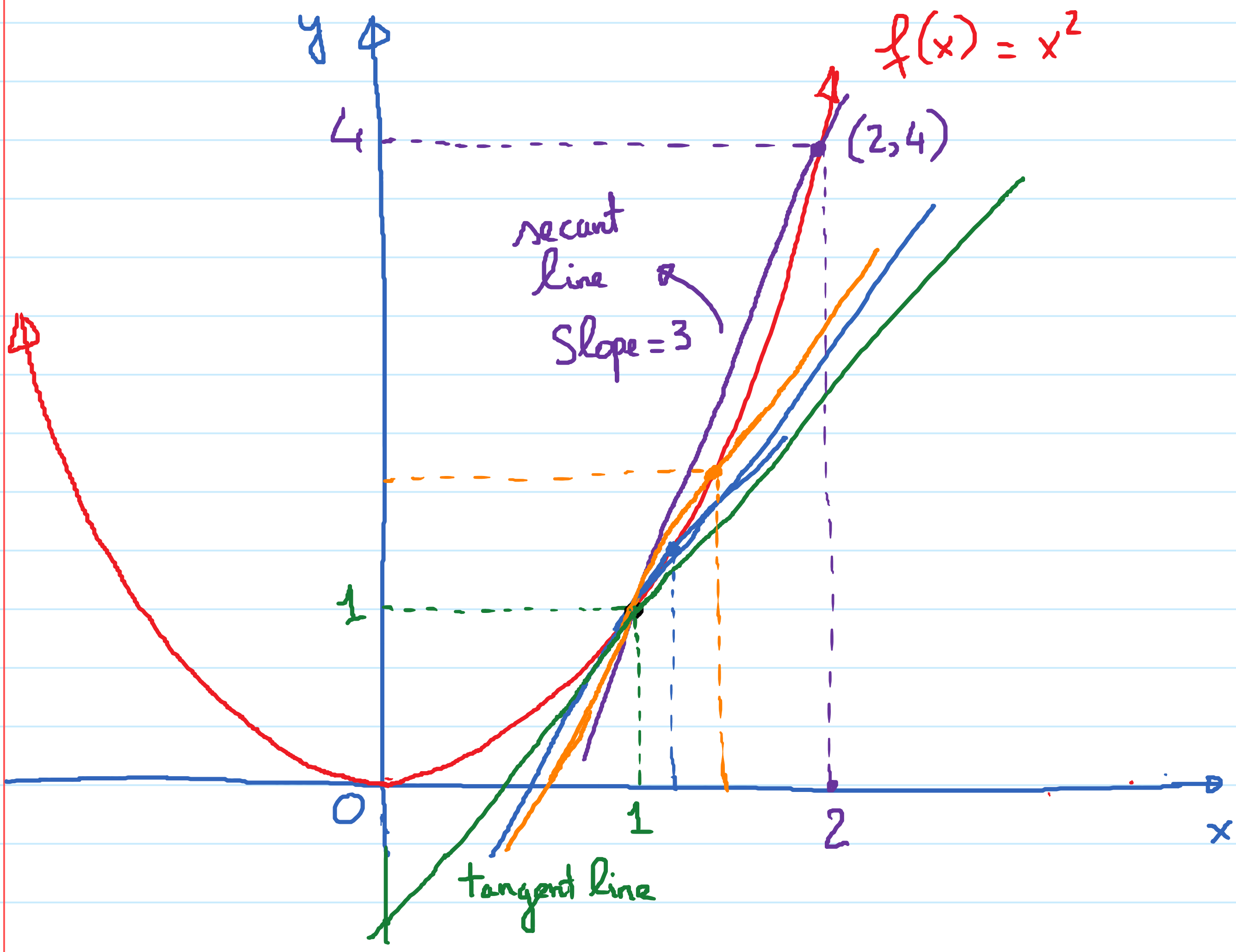
Goals: (1) The tangent line problem.

(2) The area problem

The tangent line problem

Problem: $f(x) = x^2$

Question: Find the equation of the tangent line to the graph of this function at the point $(1, 1)$



Pick $x=2$, $y=f(2)=4 \rightarrow$ point $(2,4)$

Slope of the secant line through $(1,1)$ and $(2,4)$ is

$$m_{sec} = \frac{y_2 - y_1}{x_2 - x_1} = \frac{4-1}{2-1} = \frac{3}{1} = \boxed{3}$$

Pick $x = 1.1$, $y = f(1.1) = (1.1)^2 = 1.21 \rightarrow$ point $(1.1, 1.21)$

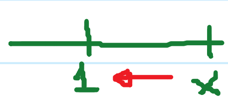
Slope of the secant line through $(1,1)$ and $(1.1, 1.21)$:

$$m_{sec} = \frac{1.21 - 1}{1.1 - 1} = \frac{0.21}{0.1} = \boxed{2.1}$$

x	$y = x^2$	$m_{sec} = \frac{x^2 - 1}{x - 1}$
2	4	3
1.1	1.21	2.1
1.01	1.0201	$\frac{1.0201 - 1}{1.01 - 1} = 2.01$
1.001	1.002001	$\frac{1.002001 - 1}{1.001 - 1} = 2.001$

Note: Here, we start with $x=2$ (on the right of 1 and we let x get close to 1 from the right

$x \rightarrow 1^+$



It appears that the slope of these secant lines get closer and closer to 2. A reasonable guess for the slope of the tangent line at $(1,1)$ is 2.

What happens to the slope of the secant line if we start with x on the left of 1?

x	$y = x^2$	$m_{sec} = \frac{x^2-1}{x-1}$
0.5	0.25	$\frac{0.25-1}{0.5-1} = 1.5$
0.9	0.81	$\frac{0.81-1}{0.9-1} = 1.9$
0.99	0.9801	$\frac{0.9801-1}{0.9-1} = 1.99$

It appears that the slopes of the secant lines also get closer and closer to 2 as x goes to 1 from the left.

$(x \longrightarrow 1^-)$

Process: Let $x \to 1^+$ and let $x \to 1^-$

and we try to figure out what the expression $\boxed{\frac{x^2-1}{x-1}}$ approaches.

Slope of secant line

→ Slope of tangent line is the "limit" of the expression $\frac{x^2-1}{x-1}$ and it is 2.

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Know: tangent line $\begin{cases} \text{Slope} = 2 \\ \text{point } (1,1) \end{cases}$

Slope-intercept equation: $y = mx + b$; $m = 2$

$$\underset{\downarrow\, 1}{y} = 2\underset{\downarrow\, 1}{x} + b \rightarrow 1 = 2\cdot 1 + b \rightarrow b = -1$$

$$\boxed{y = 2x - 1}$$

2^{nd} way of finding equation of tangent line.

Point - Slope Equation: $\boxed{y - y_1 = m(x - x_1)}$

$$y - 1 = 2(x - 1)$$

$$\rightarrow y - 1 = 2x - 2 \rightarrow \boxed{y = 2x - 1}$$

Why do people care about the tangent line problem?

In physics, $f(t) = t^2$ gives the position of an object at time t. (Position function)

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distance traveled from $t=1$ to $t=2$

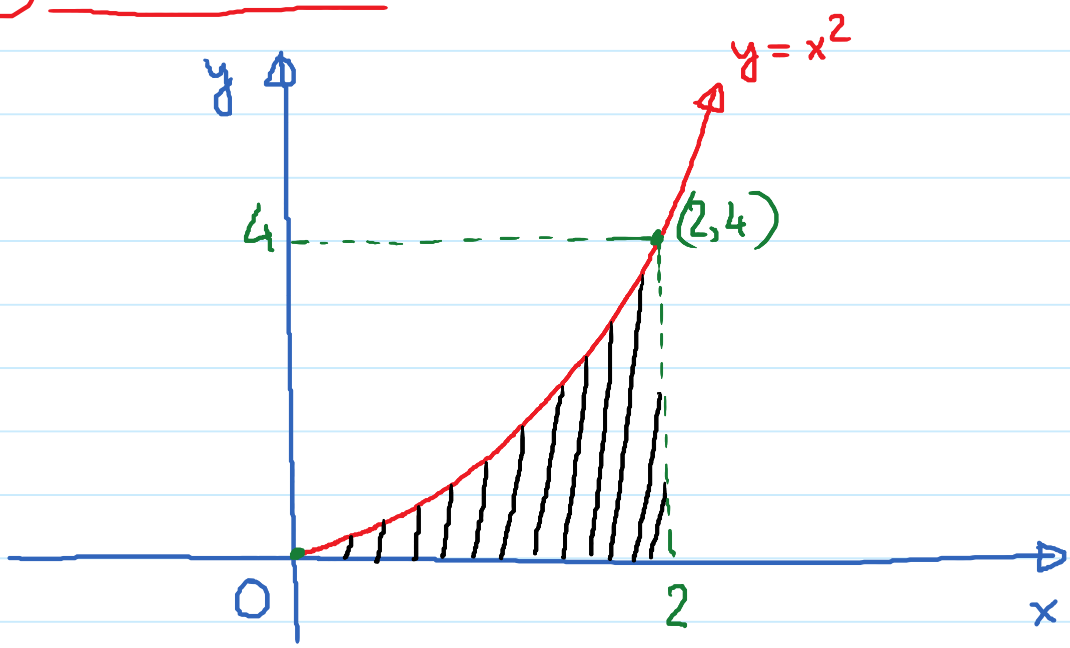
Slope of secant line through $(1,1)$, $(2,4)$ $= \dfrac{4-1}{2-1} = \dfrac{\boxed{f(2)-f(1)}}{2-1}$

time it takes to travel that distance

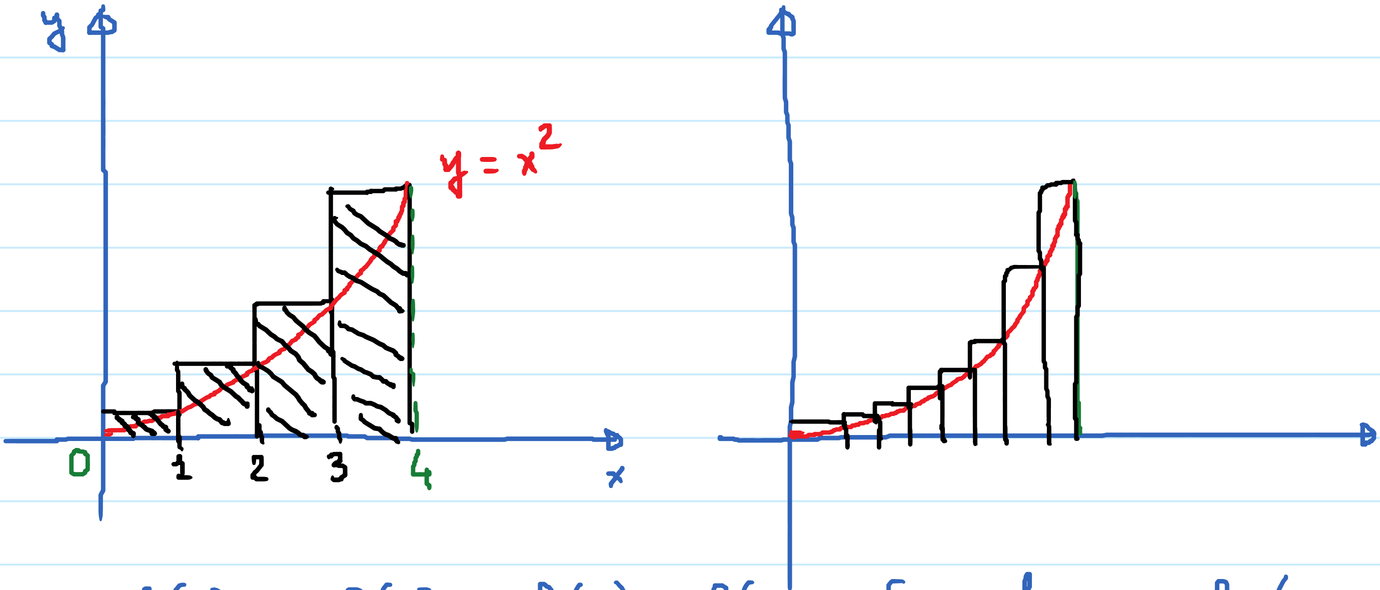
$=$ Average velocity on $[1,2]$

Slope of tangent line at $x=1$ $=$ Instantaneous velocity at $t=1$.

(2) Area Problem:



Area problem: Find the shaded area.



$f(1) + f(2) + f(3) + f(4)$ = Sum of areas of 4 rectangles

$= 1 + 4 + 9 + 16 = 30$

HW: Section 2.1