Note: Polynomial functions, Rational functions, Radical function, Log functions, Exp functions are continuous at every point in their domain

E.g. $f(x) = x^{2019} - 10x^{1000} + 4x^{500} + 7.$

Q: Determine the interval on which f is continuous?

A: $(-\infty, \infty)$

E.g. $f(x) = \dfrac{3x-5}{2x-7}$

Q: Determine the interval on which f is continuous?

A: Domain $= (-\infty, \frac{7}{2}) \cup (\frac{7}{2}, \infty)$ (interval notation)

or $\{x \mid x \neq \frac{7}{2}\}$ (set-builder notation)

f is continuous on its domain. (Rational function)

Hence, f is continuous on $(-\infty, \frac{7}{2}) \cup (\frac{7}{2}, \infty)$

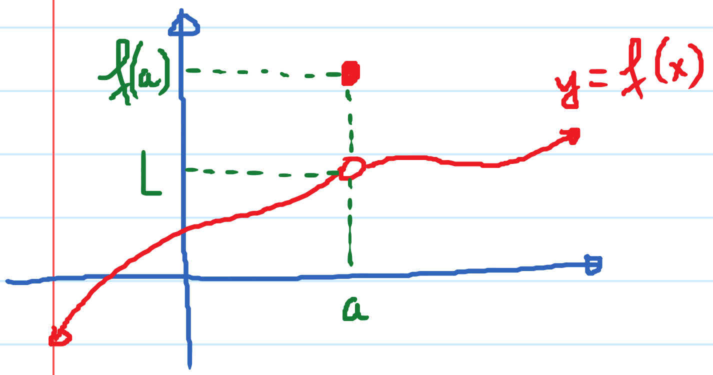
E.g. $g(x) = \dfrac{\sqrt{x-5}}{x-7}$

Q: Determine the interval on which f is continuous?

A: $[5, 7) \cup (7, \infty)$

② Classify Different types of Discontinuity.

(I) Removable Discontinuity.



We say that f has a removable discontinuity at $x = a$ if:

$\lim_{x \to a} f(x)$ exists, i.e., $\lim_{x \to a^-} f(x) = \lim_{x \to a^+} f(x) = L$

BUT: $\lim_{x \to a} f(x) \neq f(a)$

E.g. $f(x) = \begin{cases} \dfrac{x^2 + 3x + 2}{x+2} & \text{if } x \neq -2 \\ 1 & \text{if } x = -2. \end{cases}$

Claim: f has a removable discontinuity at $x = -2$.

Reason: $\lim_{x \to -2} \dfrac{x^2 + 3x + 2}{x+2} = \lim_{x \to -2} \dfrac{(x+1)\cancel{(x+2)}}{\cancel{x+2}}$

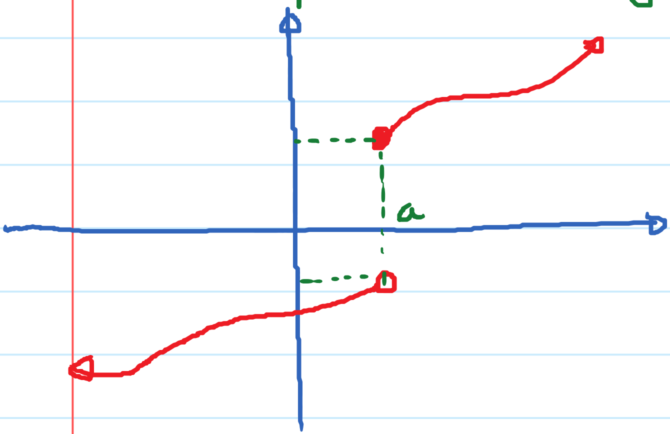
$= \lim_{x \to -2} (x+1) = -1.$

$$\underbrace{\lim_{x \to -2} f(x)}_{-1} \neq \underbrace{f(-2)}_{1}$$

limit exists but different from function value

So, f has a removable discontinuity at $x=-2$

II Jump Discontinuity.



We say that f has a jump discontinuity at $x=a$ if

$$\lim_{x \to a^-} f(x) \neq \lim_{x \to a^+} f(x).$$

E.g.

$$f(x) = \begin{cases} x\sin(x) & \text{if } x < \pi \\ x\cos(x) & \text{if } x \geq \pi \end{cases}$$

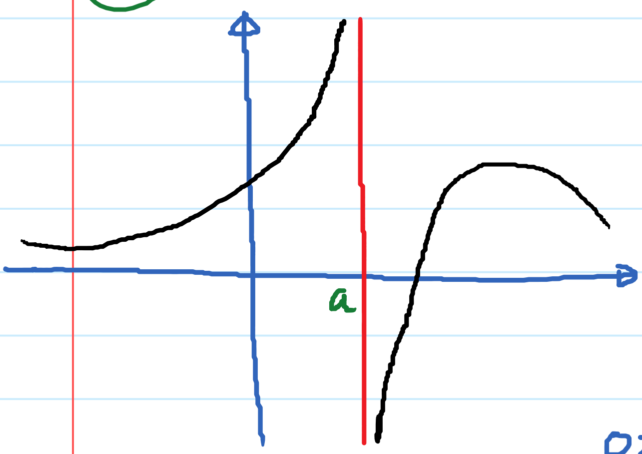
Q: Explain why f has a jump discontinuity at $x=\pi$?

Reason: $\displaystyle\lim_{x \to \pi^-} f(x) = \lim_{x \to \pi} x\sin(x) = \pi \cdot \sin(\pi) = 0$

$$\lim_{x \to \pi^+} f(x) = \lim_{x \to \pi} x\cos(x) = \pi\cos(\pi) = -\pi.$$

So, $\displaystyle\lim_{x \to \pi^-} f(x) \neq \lim_{x \to \pi^+} f(x)$. f has a jump discont. at $x=\pi$.

III Infinite Discontinuity.



We say that f has an infinite discontinuity at $x = a$ if

$$\lim_{x \to a^-} f(x) = \pm\infty$$

or

$$\lim_{x \to a^+} f(x) = \pm\infty$$

E.g.

$$f(x) = \frac{2}{(x-3)(x-4)}$$

f has infinite discontinuity at $x=3$ and at $x=4$ b/c f has V.A. at those values; hence, the limit there are infinite.