3.1. Definition of the Derivative

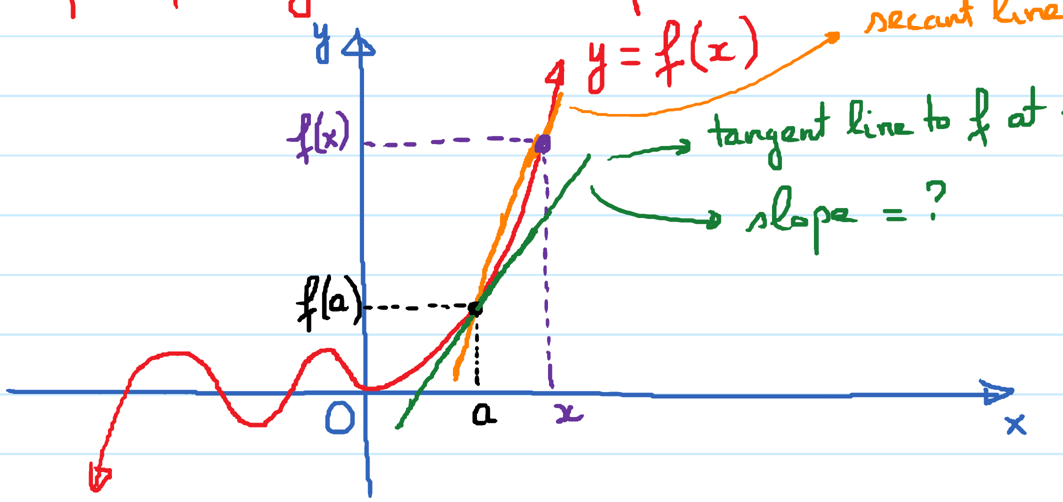
Monday, January 28, 2019 8:01 AM

Obj: (1) 2 Formulas to calculate the slope of the tangent line to the graph of a function at a point.

(2) Definition of the derivative of a function at a point

(3) Calculate derivative using definition and geometric interpretation.

(1) Slope of tangent line at a point.



Formula to calculate the slope of the tangent line to graph of $y = f(x)$ at the point where $x = a$:

(I) $$\boxed{m_{tangent} = \lim_{x \to a} \frac{f(x) - f(a)}{x - a}}$$

Where does this come from?

$$m_{sec} = \frac{f(x) - f(a)}{x - a} \; ; \; m_{tan} = \lim_{x \to a} m_{sec}$$

E.g. $f(x) = x^2$.

(a) Find slope of the tangent line to the graph of f at the point $(2,4)$.

(b) Find the slope-intercept equation of this tangent line.

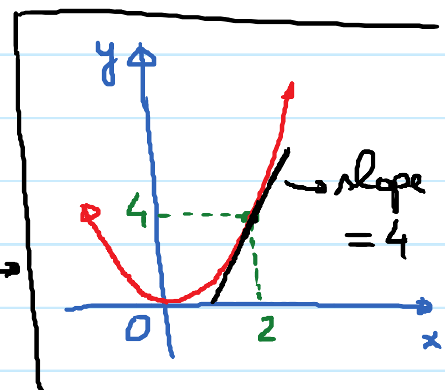
Sol: (a) $m_{\text{tangent}} = \lim_{x \to a} \frac{f(x) - f(a)}{x - a}$

$$= \lim_{x \to 2} \frac{x^2 - 4}{x - 2} \quad \left(\frac{0}{0}\right) \longrightarrow \text{more work!}$$

$$= \lim_{x \to 2} \frac{\cancel{(x-2)}(x+2)}{\cancel{x-2}}$$

$$= \lim_{x \to 2} (x+2)$$

$$m_{\text{tangent}} = 4$$



(b) Point-Slope Formula: Point $(2,4)$, slope $= 4$

$$\boxed{y - y_1 = m(x - x_1)}$$

$$y - 4 = 4(x - 2)$$

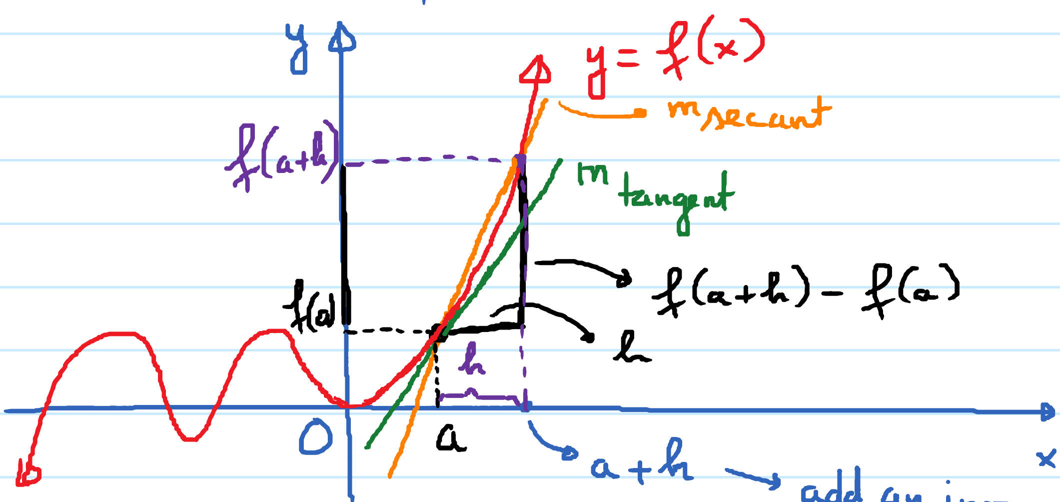
$$y - 4 = 4x - 8 \longrightarrow \boxed{y = 4x - 4}$$

Slope intercept formula.

Second (Alternative) formula to calculate the slope of the tangent line to the graph of $y = f(x)$ at $x = a$.

II
$$\boxed{m_{tangent} = \lim_{h \to 0} \frac{f(a+h) - f(a)}{h}}$$

Where does this come from?



$a + h \rightarrow$ add an increment h to a

$$m_{sec} = \frac{f(a+h) - f(a)}{h}, \quad m_{tangent} = \lim_{\boxed{h \to 0}} m_{sec}$$

let increment $\to 0$

E.g. Apply the second formula to find the slope of the tangent line to the graph of $y = \boxed{x}^2$ at $(2,4)$.

Sol: Second formula:

$$m_{tangent} = \lim_{h\to 0} \frac{f(a+h) - f(a)}{h}$$

$$f(a+h) = f(\boxed{2+h}) = (2+h)^2$$

$$m_{tangent} = \lim_{h\to 0} \frac{(2+h)^2 - 4}{h} \quad \left(\frac{0}{0}\right) \to \text{more work}$$

$$= \lim_{h\to 0} \frac{\cancel{4} + 4h + h^2 \cancel{-4}}{h}$$

$$= \lim_{h\to 0} \frac{4h + h^2}{h} = \lim_{h\to 0} \frac{\cancel{h}(4+h)}{\cancel{h}}$$

$$= \lim_{h\to 0} (4+h) = \boxed{4}$$

Remind: Square of Sum: $\boxed{(A+B)^2 = A^2 + 2AB + B^2}$

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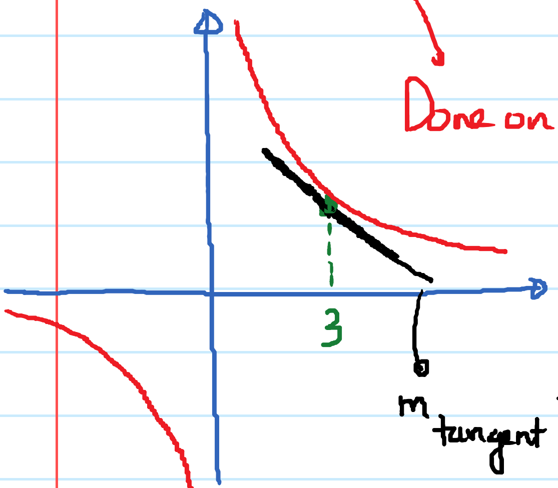
E.x. Consider $f(x) = \frac{3}{x}$

Find the slope of the tangent line to the graph of f at the point whose x-coordinate is 3.

(a) Use Formula (I) (b) Use Formula (II)

Done on the board



$f(x) = \frac{3}{\boxed{x}}$

$$m_{tangent} = \lim_{h \to 0} \frac{f(a+h) - f(a)}{h}$$

$$= \lim_{h \to 0} \frac{f(\boxed{3+h}) - f(3)}{h}$$

$$= \lim_{h \to 0} \frac{\frac{3}{3+h} - 1}{h} \left(\frac{0}{0}\right) = \lim_{h \to 0} \frac{\frac{3}{3+h} - \frac{1 \cdot (3+h)}{1 \cdot (3+h)}}{h}$$

$$= \lim_{h \to 0} \frac{\frac{3 - (3+h)}{3+h}}{h} = \lim_{h \to 0} \frac{\frac{\cancel{3} - \cancel{3} - h}{3+h}}{h}$$