$$= \lim_{h \to 0} \frac{\frac{-h}{3+h}}{\frac{h}{1}} = \lim_{h\to 0} \frac{-h}{3+h} \cdot \frac{1}{h}$$

$$= \lim_{h\to 0} \frac{-\cancel{h}}{(3+h)\cancel{h}} = \lim_{h\to \boxed{0}} \frac{-1}{3+h} = \boxed{-\frac{1}{3}}$$

Note:

The slope of the tangent line to the graph of $y = f(x)$ at a point $x = a$ is called the Derivative of f at $x = a$

So, the derivative of a function f at a point a; denoted by, $\underbrace{f'(a)}_{\text{read as } f \text{ "prime" of } a}$ (Newton) or $\underbrace{\left.\frac{dy}{dx}\right|_{x=a}}_{\text{read as } dy \text{ over } dx \text{ at } x=a}$ (Leibnitz)

and it is given by the formula:

$$f'(a) = \left.\frac{dy}{dx}\right|_{x=a} = \lim_{x\to a} \frac{f(x)-f(a)}{x-a} \quad (1^{st} \text{ formula})$$

$$= \lim_{h\to 0} \frac{f(a+h)-f(a)}{h} \quad (2^{nd} \text{ formula})$$

(provided that the limit exists)

E.g. Consider $f(x) = \sqrt{x}$.

(a) Find $f'(1)$ $\left(\text{or } \left.\frac{dy}{dx}\right|_{x=1}\right)$

(b) Find the slope-intercept equation of the tangent line to the graph of f at the point where $x = 1$.

Sol:

(a) $f'(1) = \lim_{x\to 1} \frac{f(x)-f(1)}{x-1} = \lim_{x\to 1} \frac{\sqrt{x}-1}{x-1} \left(\frac{0}{0}\right)$

$$\boxed{(A-B)(A+B) = A^2 - B^2}$$

$$= \lim_{x\to 1} \frac{\sqrt{x}-1}{x-1} \cdot \frac{\sqrt{x}+1}{\sqrt{x}+1}$$

$$= \lim_{x\to 1} \frac{\cancel{x-1}}{(\cancel{x-1})\cdot(\sqrt{x}+1)} = \lim_{x\to 1} \frac{1}{\sqrt{x}+1} = \boxed{\frac{1}{2}}$$

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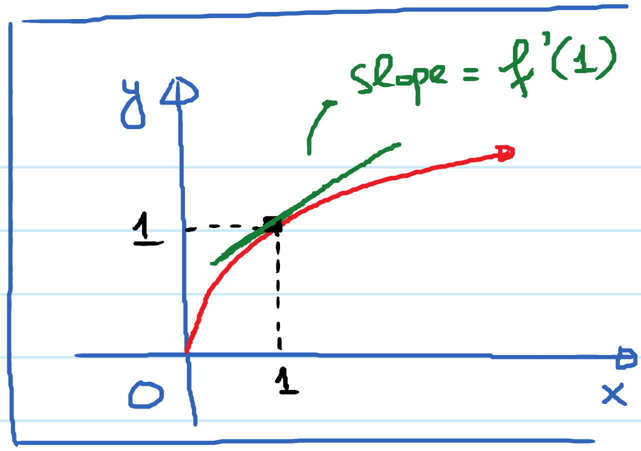
So, $f'(1) = \frac{1}{2}$



(b) Point: $(1, 1)$

$m_{tangent} = f'(1) = \frac{1}{2}$

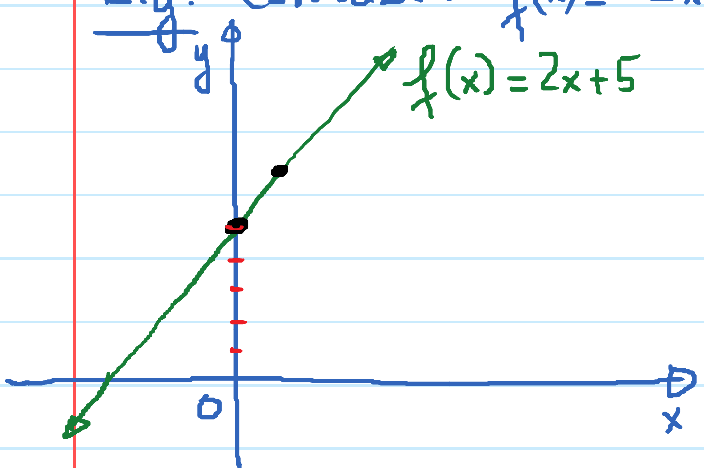
Point-Slope Formula: $y - 1 = \frac{1}{2}(x - 1)$

$$\boxed{y = \frac{1}{2}x + \frac{1}{2}}$$

Note: $f'(a)$ $\left(\text{or } \left.\frac{dy}{dx}\right|_{x=a} \right)$ = slope of the tangent line of the graph of f at point $(a, f(a))$

→ this is the geometric interpretation of the derivative of a function at a point.

E.g. Consider: $f(x) = 2x + 5$



Find $f'(2019)$ without doing any limits

Slope of tangent line to graph at any point is 2.
(b/c tangent line ≡ given line).

So, $f'(2019) = 2$.

Q: Use the limit definition (above) to find $f'(2019)$

$$f'(2019) = \lim_{h \to 0} \frac{\boxed{f(2019+h)} - f(2019)}{h}$$

$$= \lim_{h \to 0} \frac{[2 \cdot (2019+h) + 5] - 4043}{h}$$

$$= \lim_{h \to 0} \frac{\cancel{4038} + 2h \cancel{+5} - \cancel{4043}}{h}$$

$$= \lim_{h \to 0} \frac{2\cancel{h}}{\cancel{h}} = \lim_{h \to 0} 2 = \boxed{2}$$

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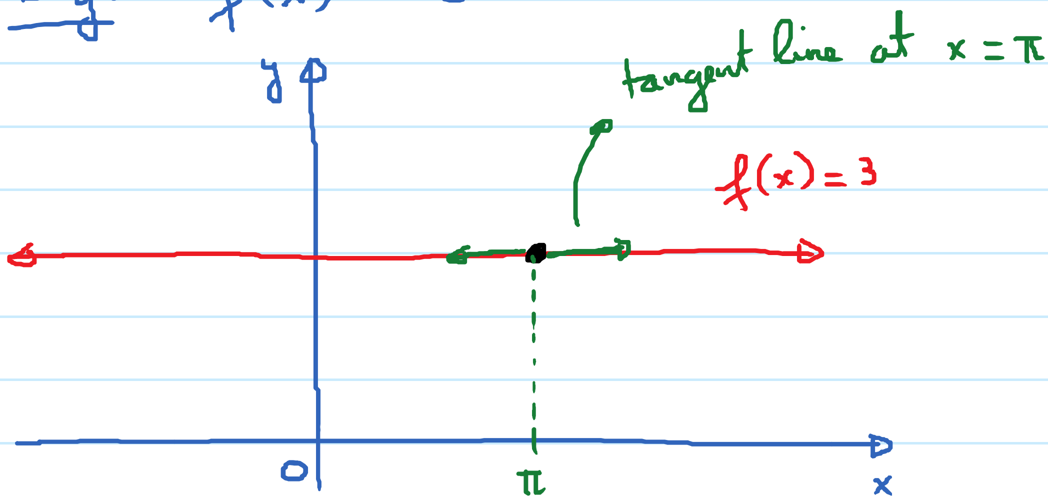
E.g. $f(x) = 3$



Find $f'(\pi)$? $f'(\pi) =$ slope tangent line $= 0$

Note: $f'(a) = \boxed{\text{instantaneous rate of change}}$ of the function f at the point $x = a$.

Why?

change in function

$$f'(a) = \lim_{x \to a} \frac{\boxed{f(x) - f(a)}}{\boxed{x - a}}$$

change in variable

instantaneous R.O.C.

Average rate of change of function