3.2. The derivative as a function

Wednesday, January 30, 2019 7:57 AM

Last time:

$$\underbrace{f'(a)}_{\text{derivative of } f \text{ at } x=a} = \text{slope of tangent line to graph of } f \text{ at } x=a$$

$$= \lim_{x \to a} \frac{f(x)-f(a)}{x-a} \qquad \text{(Formula I)}$$

$$= \lim_{h \to 0} \frac{f(a+h)-f(a)}{h} \qquad \text{(Formula II)}$$

We calculated the derivative of many functions at given points.

E.g. $f(x) = \sqrt{x}$; $f'(1) = \frac{1}{2}$

$f(x) = x^2$; $f'(2) = 4$

Today, want: given a function $y = f(x)$, develop a formula that can give us the derivative at any point

$\longrightarrow$ We look for the "derivative function" of f.

We call such a function f':

$$x \longrightarrow \boxed{\;f'\;} \longrightarrow f'(x)$$

We replace the fixed # a by the variable x in formula II:

$$\boxed{f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}}$$

This is the definition of the derivative function of a given function f. (provided that the limit exists)

Domain of $f' = \{x \mid \text{the limit exists}\}$

E.g. Given $\boxed{f(x) = x^2.}$ Find the formula of the derivative function $f'(x)$ of this function.

Sol: $$f'(x) = \lim_{h \to 0} \frac{\boxed{f(x+h)} - \boxed{f(x)}}{h}$$

$$\boxed{f(x+h) = (x+h)^2}$$

$$\boxed{f(x) = x^2}$$

We have: $$f'(x) = \lim_{h \to 0} \frac{(x+h)^2 - x^2}{h} \quad \left(\frac{0}{0}\right)$$

$$= \lim_{h \to 0} \frac{\cancel{x^2} + 2xh + h^2 \cancel{- x^2}}{h}$$

$$= \lim_{h \to 0} \frac{\cancel{h}(2x+h)}{\cancel{h}} = \lim_{h \to 0} (2x+h)$$

$$= \boxed{2x}$$

This is the formula that gives us the derivative of the function $f(x) = x^2$ at any point

Function	Derivative function	
$f(x) = x^2$	$f'(x) = 2x$	$f'(2) = 4$, $f'(100) = 200$
		$f'(-5) = -10$
		$f'(\frac{1}{2}) = 1$

Find the equation of the tangent line to the graph of $f(x) = x^2$ at the point $(10, 100)$.

Slope of tangent line at $(10, 100) = f'(10) = 20$

Pt-Slope formula: $y - 100 = 20(x - 10)$

$$\boxed{y = 20x - 100}$$

Ex.

Given $f(x) = x^3$. Use the definition of the derivative function to develop the formula for the derivative function $f'(x)$ of this.

$$f'(x) = \lim_{h\to 0} \frac{f(x+h) - f(x)}{h}$$

$$f(x+h) = (x+h)^3 \; ; \; f(x) = x^3$$

So,

$$f'(x) = \lim_{h\to 0} \frac{(x+h)^3 - x^3}{h}$$

$$= \lim_{h\to 0} \frac{\cancel{x^3} + 3x^2h + 3xh^2 + h^3 - \cancel{x^3}}{h}$$

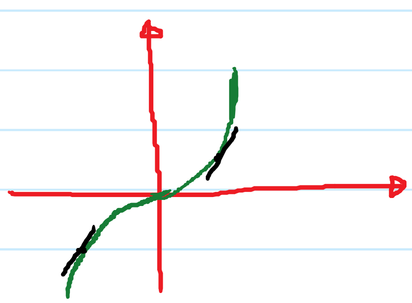
$$= \lim_{h\to 0} \frac{\cancel{h}(3x^2 + 3xh + h^2)}{\cancel{h}}$$

$$= \lim_{h\to 0} (3x^2 + 3xh + h^2)$$

$$= \boxed{3x^2}$$

So, if $f(x) = x^3$, then the derivative function of it is $f'(x) = 3x^2$.

For e.g., $f'(2) = 12$; $f'(-1) = 3$ ....



So far,

Function $f(x)$	Derivative $f'(x)$
$f(x) = x^2$	$f'(x) = 2x$
$f(x) = x^3$	$f'(x) = 3x^2$
$f(x) = x^4$	Guess: $f'(x) = 4x^3$
$f(x) = x^5$	Guess: $f'(x) = 5x^4$

→ we will have a rule for this next time.

E.g. Given $f(x) = \frac{1}{x}$.

(a) Find the formula of the derivative function $f'(x)$

(b) Find the domain of $f'(x)$

(c) Find the equation of the tangent line to the graph of f at the point where $x = -2$.