(a) Use the limit definition:

$$\boxed{f'(x) = -\frac{1}{x^2}}$$

(b) Domain of $f'(x)$ = all real #s except for 0

$= (-\infty, 0) \cup (0, \infty)$

(c) Slope $= f'(-2) = -\frac{1}{4}$

y of Point (of tangency) $= -\frac{1}{2}$ (Plug $x = -2$ into $f(x) = \frac{1}{x}$)

$\rightarrow$ Point $\left(-2, -\frac{1}{2}\right)$

Equation: $y - \left(-\frac{1}{2}\right) = -\frac{1}{4}\left(x - (-2)\right)$

$$y + \frac{1}{2} = -\frac{1}{4}(x+2)$$

$$\boxed{y = -\frac{1}{4}x - 1}$$

Pattern of derivative

function	derivative function
x^2	$2x$
x^3	$3x^2$
$\frac{1}{x} = x^{-1}$	$-1x^{-2} = -\frac{1}{x^2}$
$\frac{1}{x^2} = x^{-2}$	$-2x^{-3} = -\frac{2}{x^3}$

E.g. Given $f(x) = \sqrt{x}$. Find $f'(x)$

$$f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$$

$$= \lim_{h \to 0} \frac{\boxed{\left(\sqrt{x+h} - \sqrt{x}\right)\left(\sqrt{x+h} + \sqrt{x}\right)}}{h \cdot \left(\sqrt{x+h} + \sqrt{x}\right)}$$

$$= \lim_{h \to 0} \frac{\cancel{x} + h - \cancel{x}}{h \cdot \left(\sqrt{x+h} + \sqrt{x}\right)}$$

$$= \lim_{h \to 0} \frac{\cancel{h}}{\cancel{h}\left(\sqrt{x+h} + \sqrt{x}\right)}$$

$$= \lim_{h \to 0} \frac{1}{\sqrt{x+h} + \sqrt{x}} = \frac{1}{\sqrt{x} + \sqrt{x}} = \boxed{\frac{1}{2\sqrt{x}}}$$

So, $f'(x) = \dfrac{1}{2\sqrt{x}}$. (Note: Domain $= (0, \infty)$)

$f'(x)$ DNE at $x = 0$, or any negative #

Differentiability:

We say that a function $y = f(x)$ is differentiable at a point x if $f'(x)$ exists at that point.

E.g. $f(x) = x^2$ is differentiable everywhere.

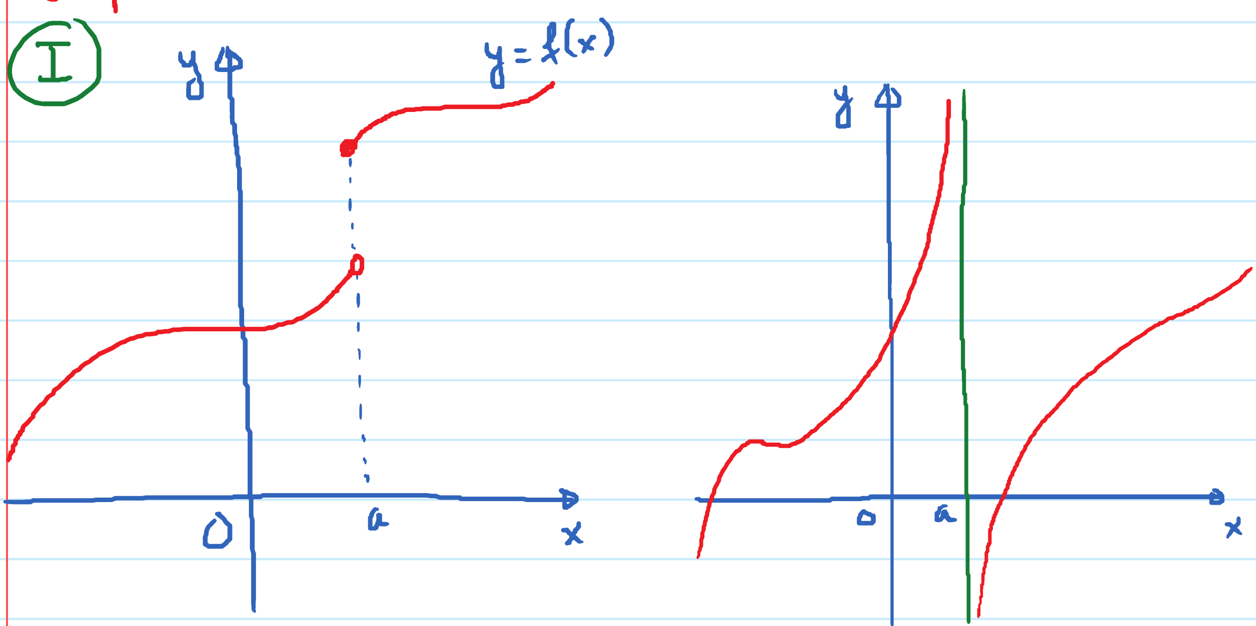
$f(x) = \frac{1}{x}$ is differentiable everywhere except at $x=0$

$f(x) = \sqrt{x}$. is differentiable on $(0, \infty)$

is NOT differentiable on $(-\infty, 0]$.

How to detect differentiability of a function given its graph?

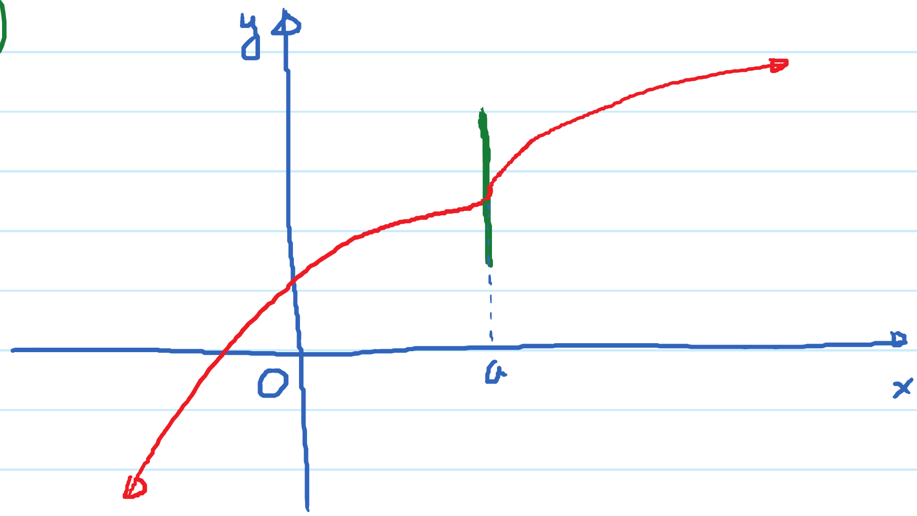
I



If f is NOT continuous at a point $x=a$, then f will NOT be differentiable at $x=a$

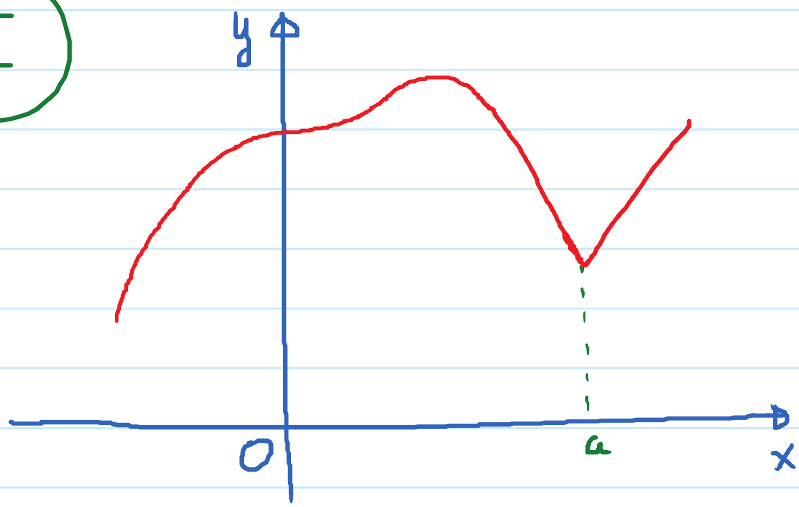
Wednesday, January 30, 2019 9:32 AM

II



If f has a vertical tangent line at $x=a$, then f is NOT differentiable at $x=a$

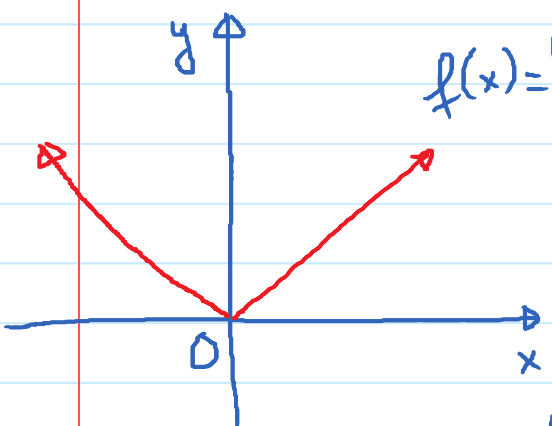
III



If f has a sharp corner at $x=a$, then f is NOT differentiable at $x=a$.

Why? We will demonstrate that for the function $f(x)=|x|$, the derivative DNE at $x=0$.

In other words, $f(x) = |x|$ is NOT differentiable at $x = 0$



$$f(x) = |x| = \begin{cases} x & \text{if } x \geq 0 \\ -x & \text{if } x < 0 \end{cases}$$

$$f'(0) = \lim_{h \to 0} \frac{f(0+h) - f(0)}{h}$$

$$= \lim_{h \to 0} \frac{f(h) - f(0)}{h}$$

$$= \lim_{h \to 0} \frac{|h| - 0}{h} = \lim_{h \to 0} \frac{|h|}{h}$$

$$\lim_{h \to 0^+} \frac{h}{h} = 1 \quad ; \quad \lim_{h \to 0^-} \frac{(-h)}{h} = -1$$

Left limit $\neq$ Right limit $\rightarrow$ limit DNE $\rightarrow$ $f'(0)$ DNE