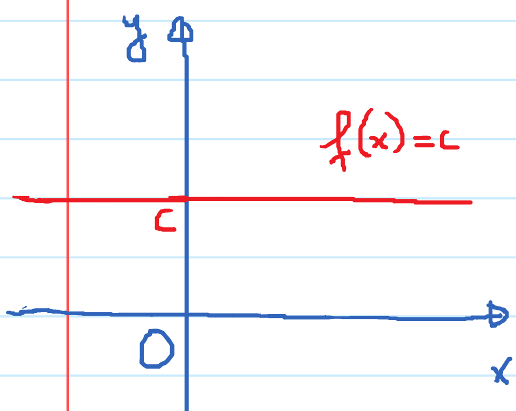
3.3. Differentiation Rules

Monday, February 4, 2019 8:06 AM

Goals:
1. Power Rule
2. Sum and Difference Rule
3. Product and Quotient Rule

* Derivative of a constant function:

Given the function $f(x) = c$ where c is a constant.



Derivative function:

$f'(x) = 0$ (Slope of tangent line at any point $= 0$)

In Leibnitz notation: $\dfrac{d}{dx}[c] = 0$

read as d over dx of c and it means we take the derivative w.r.t. x of the constant function $y=c$.

1. Power Rule

$$\frac{d}{dx}\left[x^2\right] = 2x \; ; \; \frac{d}{dx}\left[x^3\right] = 3x^2 \; ; \; \frac{d}{dx}\left[x^{-1}\right] = -1x^{-2} = \frac{-1}{x^2}$$

In general,

$$\boxed{\frac{d}{dx}\left[x^n\right] = n \cdot x^{n-1}}$$; n is an integer.

It turns out that this formula is true when the exponent is any real #,

$$\boxed{\frac{d}{dx}\left[x^n\right] = n \cdot x^{n-1}}$$; n is any real #

In Newton's notation:

If $f(x) = x^n$, then $f'(x) = n \cdot x^{n-1}$

Or in short, $\left(x^n\right)' = n \cdot x^{n-1}$

$$x^{-n} = \frac{1}{x^n}$$

E.g. Find the given derivative.

(a) $\frac{d}{dx}\left[x^1\right] = 1 \cdot x^{1-1} = 1 \cdot x^0 = 1.$

(b) $\frac{d}{dx}\left[\sqrt{x}\right] = \frac{d}{dx}\left[x^{\boxed{\frac{1}{2}}}\right] = \frac{1}{2} \cdot x^{-\frac{1}{2}} = \frac{1}{2x^{1/2}} = \frac{1}{2\sqrt{x}}$

rewrite

$$\boxed{\frac{d}{dx}\left[\sqrt{x}\right] = \frac{1}{2\sqrt{x}} \quad \text{or} \quad \left(\sqrt{x}\right)' = \frac{1}{2\sqrt{x}}}$$

(c) $$\frac{d}{dx}\left[\frac{1}{x}\right] = \frac{d}{dx}\left[x^{\boxed{-1}}\right] = -1\cdot x^{-2} = \frac{-1}{x^2}$$

$$\boxed{\frac{d}{dx}\left[\frac{1}{x}\right] = -\frac{1}{x^2} \quad \text{or} \quad \left(\frac{1}{x}\right)' = -\frac{1}{x^2}}$$

(d) $$\frac{d}{dx}\left[\sqrt[5]{x^7}\right] = \frac{d}{dx}\left[x^{\boxed{\frac{7}{5}}}\right] = \frac{7}{5}x^{\frac{2}{5}} = \frac{7}{5}\sqrt[5]{x^2}$$

Reminder:

$$x^{\frac{m}{n}} \quad \begin{cases} \sqrt[n]{x^m} \\ \left(\sqrt[n]{x}\right)^m \end{cases}$$

Ex: Find the derivative:

(a) $\frac{d}{dx}\left[x^{\pi}\right]$ ✓ (b) $\frac{d}{dx}\left[e^{2019}\right] = 0$ (c) $\frac{d}{dx}\left[\sqrt{2}\right] = 0$

(d) $\frac{d}{dx}\left[\sqrt[3]{x}\right]$ (e) $\frac{d}{dx}\left[\frac{1}{\sqrt[5]{x^7}}\right] = -\frac{7}{5}x^{-\frac{12}{5}}$

Equivalent

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d) $\frac{d}{dx}\left[x^{\boxed{\frac{1}{3}}}\right] = \frac{1}{3}x^{\frac{1}{3}-1} = \boxed{\frac{1}{3}x^{-\frac{2}{3}} = \frac{1}{3x^{2/3}} = \frac{1}{3\sqrt[3]{x^2}}}$

* Constant multiple Rule:

$\frac{d}{dx}\left[x^2\right] = 2x$. What about $\frac{d}{dx}\left[10x^2\right] = 10 \cdot 2x$

$= 20x$

$$\boxed{\begin{gathered}\frac{d}{dx}\left(c \cdot f(x)\right) = c \cdot \frac{d}{dx}\left[f(x)\right] \\ \text{or} \quad \left(c \cdot f(x)\right)' = c \cdot \left(f(x)\right)'\end{gathered}}$$

Sum | difference Rule:

In Leibnitz notation:

$$\frac{d}{dx}\left[f(x) \pm g(x)\right] = \frac{d}{dx}\left[f(x)\right] \pm \frac{d}{dx}\left[g(x)\right]$$

In prime notation:

$$\left(f(x) \pm g(x)\right)' = f'(x) \pm g'(x)$$

E.g. $f(x) = x^3 + 3x^2 - 20x + 2019$

Find $f'(x)$.

$$\boxed{f'(x) = 3x^2 + 6x - 20.}$$

E.g. Find $\left.\frac{d}{dx}\left[-2x^3 + 4\sqrt{x} - \frac{3}{x} + 5\right]\right|_{x=1}$ } Evaluate derivative at $x=1$

Step 1: Find derivative

$$\frac{d}{dx}\left[-2x^3 + 4\sqrt{x} - \frac{3}{x} + 5\right] = -6x^2 + \frac{2}{\sqrt{x}} + \frac{3}{x^2}$$

Step 2: Plug $x=1$ to derivative

$$-6(1)^2 + \frac{2}{\sqrt{1}} + \frac{3}{(1)^2} = -6 + 2 + 3 = \boxed{-1}$$

Ex. Find the equation of the tangent line to $y = \sqrt[4]{x}$ at $(1,1)$.

Sol: $y = \sqrt[4]{x} = x^{\frac{1}{4}}$. So, $y' = \frac{1}{4} x^{-\frac{3}{4}} = \frac{1}{4x^{3/4}}$