

$$\text{Slope} = \frac{1}{4(1)^{3/4}} = \boxed{\frac{1}{4}}$$

Plug $x=1$ to derivative

Point-Slope Equation: $y - 1 = \frac{1}{4}(x - 1)$

$$\boxed{y = \frac{1}{4}x + \frac{3}{4}} \rightarrow \text{Slope-intercept Equation.}$$

E.g. $f(x) = x^3 - 4x^2 + 3x + 6$.

Q: Find all the x -values at which the graph has a horizontal tangent line.

Process: Take derivative and set it $= 0$ and solve for x .

Sol: $f'(x) = 3x^2 - 8x + 3 = 0$

Quadratic Formula: $\frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

$$x = \frac{8 \pm \sqrt{64 - 36}}{6} = \frac{8 \pm \sqrt{28}}{6} = \frac{8 \pm 2\sqrt{7}}{6} = \frac{4 \pm \sqrt{7}}{3}$$

Answer: $x = \frac{4+\sqrt{7}}{3}$; $x = \frac{4-\sqrt{7}}{3}$. (horizontal tangent line at these locations)

Product Rule:

~~$[f(x) \cdot g(x)]' = f'(x) \cdot g'(x)$ WRONG.~~

Correct formula for the product rule:

In "prime" notation:

$$[f(x) \cdot g(x)]' = f'(x) \cdot g(x) + f(x) \cdot g'(x)$$

In Leibnitz notation: set $u = f(x)$; $v = g(x)$

$$\frac{d}{dx}[u \cdot v] = v \cdot \frac{du}{dx} + u \cdot \frac{dv}{dx}$$

Quotient Rule:

~~$\left(\frac{f(x)}{g(x)}\right)' = \frac{f'(x)}{g'(x)}$ Wrong~~

Correct formula for quotient rule:

In "prime" notation:

$$\left(\frac{f(x)}{g(x)} \right)' = \frac{f'(x) \cdot g(x) - f(x) \cdot g'(x)}{(g(x))^2}$$

In Leibnitz notation: set $f(x) = u$; $g(x) = v$

$$\frac{d}{dx} \left[\frac{u}{v} \right] = \frac{v \cdot \frac{du}{dx} - u \cdot \frac{dv}{dx}}{v^2}$$

$$\frac{d}{dx} \left[\frac{u}{v} \right] = \frac{\overset{\text{low}}{v} \cdot \overset{\text{dhigh}}{\frac{du}{dx}} - \overset{\text{high}}{u} \cdot \overset{\text{dlow}}{\frac{dv}{dx}}}{v^2 (\text{low})^2}$$

E.g. $h(x) = \frac{3x+1}{4x-3}$. Find $h'(x)$.

$$h'(x) = \frac{3 \cdot (4x-3) - 4 \cdot (3x+1)}{(4x-3)^2} = \frac{12x-9-12x-4}{(4x-3)^2}$$

$$h'(x) = - \frac{13}{(4x-3)^2}$$

E.g.:

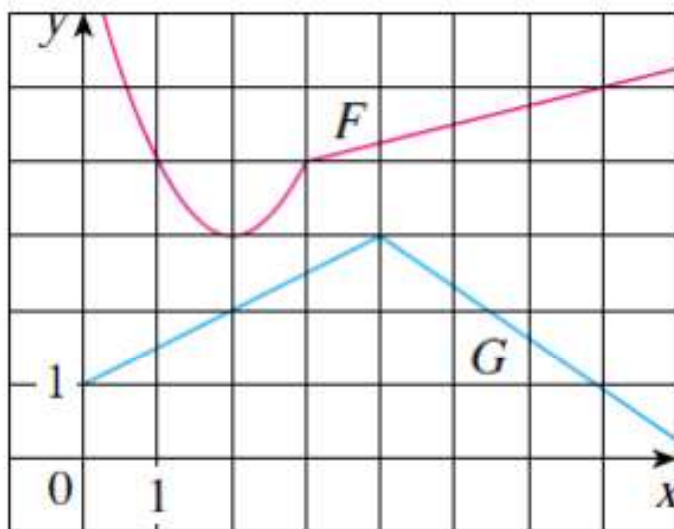
$$w(x) = \frac{1 - x^8}{1 + x^8} \quad \text{Find } w'(x).$$

$$w'(x) = \frac{-8x^7 \cdot (1 + x^8) - 8x^7 \cdot (1 - x^8)}{(1 + x^8)^2}$$

quotient rule

$$= \frac{-8x^7 - \cancel{8x^{15}} - 8x^7 + \cancel{8x^{15}}}{(1 + x^8)^2}$$

$$w'(x) = \frac{-16x^7}{(1 + x^8)^2}$$



$$H(x) = \frac{F(x)}{G(x)}$$

Find $H'(7)$?

$$W(x) = F(x)G(x)$$

$$W'(2) = ?$$

Higher order derivative.

$$f(x) = x^{10}$$

$$f'(x) = 10x^9 \rightarrow \text{first derivative.}$$

$$f''(x) = 90x^8 \rightarrow \text{second derivative.}$$

$$f'''(x) = 720x^7 \rightarrow \text{third derivative}$$

$$f^{(4)}(x) = 5040x^6 \rightarrow \text{fourth derivative.}$$

⋮

$$f^{(11)}(x) = 0 ; \quad f^{(n)}(x) = 0 \text{ for all } n \geq 11$$

$\underbrace{\hspace{1cm}}$
nth order derivative of f .

$$f^{(n)}(x) = \text{derivative of } f^{(n-1)}(x)$$

In Leibnitz notation: 1st derivative: $\frac{dy}{dx}$

2nd derivative: $\frac{d^2 y}{dx^2}$; 3rd derivative $\frac{d^3 y}{dx^3}$

nth derivative: $\frac{d^n y}{dx^n}$