3.4. Derivatives and Rates of Change

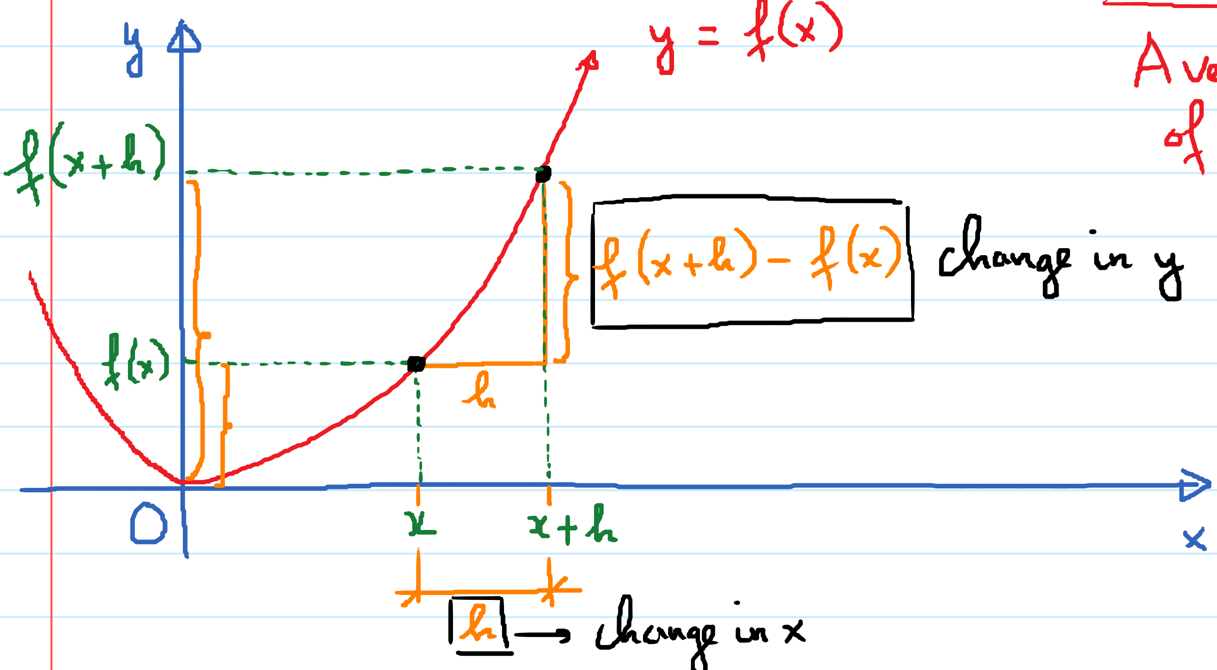
Wednesday, February 13, 2019 8:04 AM

Key idea: $\boxed{f'(x) = \text{Instantaneous Rate of Change of } f \text{ at } x}$

Why?

$$f'(x) \overset{\text{def}}{=} \lim_{h \to 0} \boxed{\frac{f(x+h) - f(x)}{h}} \qquad \boxed{\frac{\text{Change in } y}{\text{change in } x}}$$

Average Rate of change of f



$$\lim_{h \to 0} \frac{\text{change in } y}{\text{change in } x} = \text{instantaneous R.O.C.}$$

In physics, $y = f(t)$ = position function of an object.

Then $f'(t)$ = instantaneous R.O.C. of position

= instantaneous velocity.

$$\boxed{v(t) = f'(t) = \text{velocity function}}$$

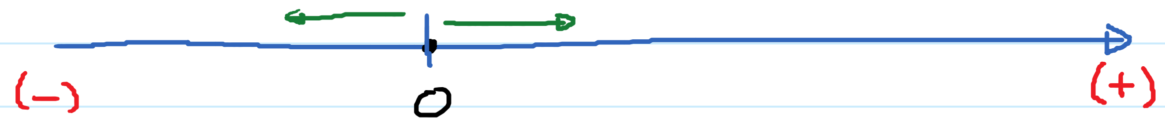
$v'(t)$ = instantaneous rate of change of velocity

= instantaneous acceleration.

$$\boxed{a(t) = v'(t) = f''(t) = \text{acceleration function}}$$

E.g. The position function of a particle moving along an axis is given by the formula:

$$\boxed{s(t) = t^3 - 9t^2 + 24t + 4 \; ; \; t \geq 0}$$ (position function)



ⓐ At what time(s) is the particle $\underbrace{\text{at rest}}_{\text{means velocity } = 0}$?

$v(t) = s'(t) = 3t^2 - 18t + 24$ (velocity function)

$v(t) = 0$: $3t^2 - 18t + 24 = 0$

$$(t-4)(3t-6) = 0$$

$$3(t-4)(t-2) = 0$$

$\boxed{t=4}$ (seconds) or $\boxed{t=2}$ (seconds)

(b) During which time interval(s) is the particle moving from $\underbrace{\text{left to right}}_{\text{move in (+) direction}}$? from $\underbrace{\text{right to left}}_{\text{move in (−) direction}}$?

move in (+) direction $\equiv$ $\boxed{v > 0}$

move in (−) direction $\equiv$ $\boxed{v < 0}$

$v(t) = \boxed{3(t-4)(t-2)}$ > 0 / < 0

→ Solve 2 inequalities

$\boxed{t=1}$ $t=2$ $\boxed{t=3}$ $t=4$ Test point $\boxed{t=5}$

0   (+) $(v>0)$ $v=0$ (−) $(v<0)$ $v=0$ (+) $(v>0)$

<u>Conclusion:</u> Particle moving right to left on $\boxed{(2,4)}$

Particle moving left to right on $\boxed{(0,2) \cup (4,\infty)}$

(c) During which time interval(s) is the particle speeding up | slowing down ?

Speed up $\equiv$ acceleration and velocity have the same sign.

$$\boxed{\begin{cases} a > 0 \\ v > 0 \end{cases} \quad \text{or} \quad \begin{cases} a < 0 \\ v < 0 \end{cases}}$$

Slow down = acceleration and velocity have different signs.

$$\boxed{\begin{cases} a > 0 \\ v < 0 \end{cases} \quad \text{or} \quad \begin{cases} a < 0 \\ v > 0 \end{cases}}$$

$a(t) = v'(t) = 6t - 18$ (acceleration function)

t	0		2		3		4	
v		+	0	−		−	0	+
a		−		−	0	+		+
		Slow down		Speed up		Slow down		Speed up

Conclusion:

Speed up on: $(2,3) \cup (4,\infty)$

Slow down on: $(0,2) \cup (3,4)$

HW #15: Position function: $s(t) = \dfrac{t}{4+t^2}$; $t \geq 0$.

(a) Velocity function:

quotient rule

$$v(t) = s'(t) = \frac{4-t^2}{(4+t^2)^2} = \boxed{\frac{4-t^2}{16+8t^2+t^4}}$$

(b) Acceleration function:

$$a(t) = v'(t) = \frac{-2t(16+8t^2+t^4) - (16t+4t^3)(4-t^2)}{(4+t^2)^4}$$

$$= \frac{-2t(4+t^2)^2 - 4t(4+t^2)(4-t^2)}{(4+t^2)^4}$$

$$= \frac{-2t\cancel{(4+t^2)}\left[4+t^2+2(4-t^2)\right]}{(4+t^2)^{\cancel{4}\,3}}$$

$$\boxed{a(t) = \frac{-2t(12-t^2)}{(4+t^2)^3} = \frac{2t(t^2-12)}{(4+t^2)^3}}$$

(b) Time interval(s) — Slow down / Speed up

t at which $v(t) = 0$

$$v(t) = \frac{4-t^2}{(4+t^2)^2} = 0 \leftrightarrow 4 - t^2 = 0$$

$$\leftrightarrow t = \pm 2$$

$t \geq 0$ (Given) $\longrightarrow$ $\boxed{t = 2.}$

t at which $a(t) = 0$

$$a(t) = \frac{2t(t^2-12)}{(4+t^2)^3} = 0 \leftrightarrow 2t(t^2-12) = 0$$

$$\longrightarrow t = 0 \quad \text{or} \quad t^2 - 12 = 0$$

$$t = \pm\sqrt{12} = \pm 2\sqrt{3}$$

$t \geq 0$ (Given) $\longrightarrow$ $\boxed{t=0}$; $\boxed{t = 2\sqrt{3}}$

t	0	$t=1$	2	$t=3$	$2\sqrt{3}$	$t=5$
$v(t)$		$+$	0	$-$		$-$
$a(t)$	0	$-$		$-$	0	$+$
		Slow down		Speed up		Slow down