3.5. Derivatives of Trigonometric Functions

Wednesday, February 6, 2019 8:05 AM

Scantron: Form # 19639. (Quiz Sheet A)

Reminder: Differentiation Rules:

$$\frac{d}{dx}\left[x^n\right] = n \cdot x^{n-1} \quad \text{(Power Rule)}$$

u, v are functions of x

$$\frac{d}{dx}\left[u \pm v\right] = \frac{du}{dx} \pm \frac{dv}{dx} \quad \text{(Sum / Difference)}$$

$$\frac{d}{dx}\left[c \cdot u\right] = c \cdot \frac{du}{dx} \quad \text{(Constant Multiple)}$$

$$\frac{d}{dx}\left[u \cdot v\right] = v\frac{du}{dx} + u\frac{dv}{dx} \quad \text{(Product)}$$

$$\frac{d}{dx}\left[\frac{u}{v}\right] = \frac{v\frac{du}{dx} - u \cdot \frac{dv}{dx}}{v^2} \quad \text{(Quotient)}$$

$$\frac{d}{dx}\left[\sqrt{x}\right] = \frac{1}{2\sqrt{x}} \quad ; \quad \frac{d}{dx}\left[\frac{1}{x}\right] = -\frac{1}{x^2}$$

Goal: Develop formulas to find the derivative of trig functions: $\sin x$, $\cos x$, $\sec x$, $\csc x$, $\tan x$, $\cot x$

Reminder of Basic Trig Identities.

$$\sec(x) = \frac{1}{\cos x} \quad ; \quad \csc(x) = \frac{1}{\sin x}$$

$$\tan x = \frac{\sin x}{\cos x} \quad ; \quad \cot x = \frac{\cos x}{\sin x}$$

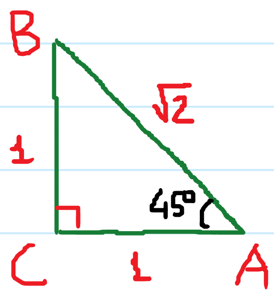
Reciprocal Identities.

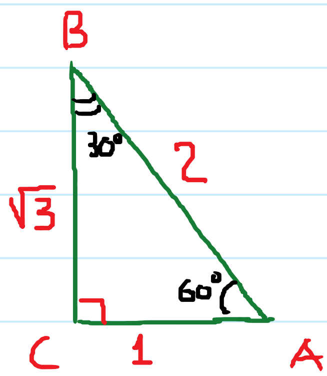
$$\sin^2 x + \cos^2 x = 1$$

$$\tan^2 x + 1 = \sec^2 x$$

$$\cot^2 x + 1 = \csc^2 x$$

Pythagorean Identities





α, β: angles

$$\sin(\alpha \pm \beta) = \sin\alpha \cos\beta \pm \cos\alpha \sin\beta$$

$$\cos(\alpha \pm \beta) = \cos\alpha \cos\beta \mp \sin\alpha \sin\beta$$

* Develop the formula for the derivative function of $f(x) = \sin x$

limit definition

$$f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$$

$$= \lim_{h \to 0} \frac{\sin(x+h) - \sin x}{h}$$

Sine of a sum

$$= \lim_{h \to 0} \frac{\sin x \cos h + \cos x \sin h - \sin x}{h}$$

$$= \lim_{h \to 0} \frac{\sin x\,[\cos h - 1]}{h} + \lim_{h \to 0} \frac{\cos x \sin h}{h}$$

$$= \sin x \underbrace{\boxed{\lim_{h \to 0} \frac{\cos h - 1}{h}}}_{0} + \cos x \underbrace{\boxed{\lim_{h \to 0} \frac{\sin h}{h}}}_{1}$$

$$= \sin x \cdot 0 + \cos x \cdot 1$$

$$\boxed{f'(x) = \cos x}$$

If $f(x) = \sin x$, then $f'(x) = \cos x$.

In "prime" notation:

$$\boxed{(\sin x)' = \cos x}$$

In Leibnitz notation:

$$\boxed{\frac{d}{dx}[\sin x] = \cos x}$$

With a similar derivation, we can obtain:

$$\boxed{(\cos x)' = -\sin x}$$

or

$$\boxed{\frac{d}{dx}[\cos x] = -\sin x}$$

* Develop the formula for the derivative of $f(x) = \tan x$

$$f(x) = \tan x = \frac{\sin x}{\cos x}$$

By the quotient rule:

$$f'(x) = \frac{\cos x \cdot \cos x - \sin x \cdot (-\sin x)}{\cos^2 x} = \frac{\boxed{\cos^2 x + \sin^2 x}^{\,1}}{\cos^2 x}$$

$$= \sec^2 x$$

$$\boxed{(\tan x)' = \sec^2 x}$$

or

$$\boxed{\frac{d}{dx}[\tan x] = \sec^2 x}$$

Similarly,

$$\boxed{(\cot x)' = -\csc^2 x}$$

or

$$\boxed{\frac{d}{dx}[\cot x] = -\csc^2 x}$$

* Develop the formula for $f(x) = \csc x = \dfrac{1}{\sin x}$

$$f'(x) = \frac{0 \cdot \sin x - 1 \cdot \cos x}{\sin^2 x} = \frac{-\cos x}{\sin^2 x}$$

$$= -\frac{\cos x}{\sin x} \cdot \frac{1}{\sin x} = -\cot x \cdot \csc x .$$

So,

$$\boxed{(\csc x)' = -\cot x \cdot \csc x}$$

or

$$\boxed{\frac{d}{dx}[\csc x] = -\cot x \cdot \csc x}$$

Similarly, $\boxed{(\sec x)' = \tan x \cdot \sec x}$

or $\boxed{\frac{d}{dx}\left[\sec x\right] = \tan x \cdot \sec x}$

To sum up,

$(\sin x)' = \cos x$	$(\cot x)' = -\csc^2 x$
$(\cos x)' = -\sin x$	$(\sec x)' = \sec x \cdot \tan x$
$(\tan x)' = \sec^2 x$	$(\csc x)' = -\csc x \cdot \cot x$

E.g. $f(x) = 4\sin x + 6\cos x$

Tangent line at $x = 0$?

$f'(x) = 4\cos x - 6\sin x$

$f'(0) = 4\cos(0) - 6\sin(0) = \boxed{4} \rightarrow$ slope.

Point $\left(x = 0 ;\ y = 4\sin(0) + 6\cos(0) = 6\right)$

$(0; 6)$

Equation: $\boxed{y = 4x + 6}$

Wednesday, February 6, 2019 9:09 AM

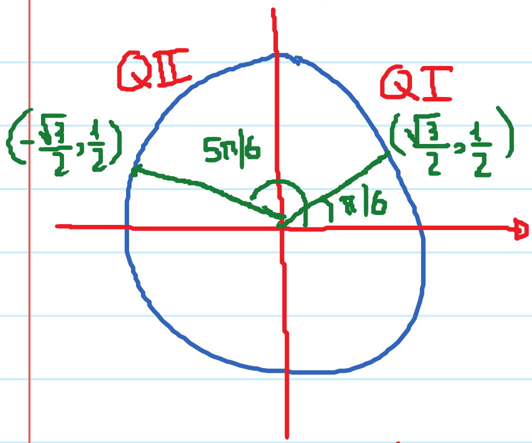
E.g. $f(x) = x - 2\cos x$

Slope $= 2$?

$f'(x) = 1 + 2\sin x$.

Set $f'(x) = 2$: $1 + 2\sin x = 2$

$2\sin x = 1 \longrightarrow \sin x = \frac{1}{2}$



$\boxed{x = \frac{\pi}{6}}$ or $\boxed{x = \frac{5\pi}{6}}$

Did HW and Review sheet on the board.