$f(x) = \tan x \quad ; \quad f^{-1}(x) = \arctan x.$

By I.F.T,

$$(f^{-1})'(x) = \frac{1}{f'(f^{-1}(x))}$$

We have: $f'(x) = \sec^2 x.$

So, $f'(f^{-1}(x)) = \sec^2(\arctan x)$

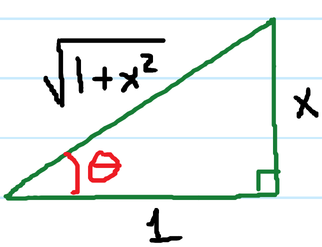
Hence,

$$(f^{-1})'(x) = \frac{1}{\sec^2(\arctan x)}$$

Now, we will simplify: $\sec^2(\arctan x).$

Let's simplify $\sec(\underbrace{\boxed{\arctan x}}_{\theta})$

Let $\theta = \arctan x$. Then $\tan\theta = x$



So, $\sec(\arctan x) = \sec\theta = \sqrt{1+x^2}$

Hence, $\sec^2(\arctan x) = (\sqrt{1+x^2})^2 = \boxed{1+x^2}$

Thus, $(f^{-1})'(x) = \dfrac{1}{1+x^2}.$

Summary of derivatives of Inverse Trig functions:

$$\left(\arcsin x\right)' = \frac{1}{\sqrt{1-x^2}} \; ; \; \left(\arccos x\right)' = \frac{-1}{\sqrt{1-x^2}}$$

$$\left(\arctan x\right)' = \frac{1}{1+x^2} \; ; \; \left(\operatorname{arccot} x\right)' = \frac{-1}{1+x^2}$$

$$\left(\operatorname{arcsec} x\right)' = \frac{1}{|x|\sqrt{x^2-1}} \; ; \; \left(\operatorname{arccsc} x\right)' = \frac{-1}{|x|\sqrt{x^2-1}}$$

Mixing this with chain rule: (u is a function of x)

$$\left(\arcsin u\right)' = \frac{u'}{\sqrt{1-u^2}} \; ; \; \left(\arccos u\right)' = \frac{-u'}{\sqrt{1-u^2}}$$

$$\left(\arctan u\right)' = \frac{u'}{1+u^2} \; ; \; \left(\operatorname{arccot} u\right)' = \frac{-u'}{1+u^2}$$

$$\left(\operatorname{arcsec} u\right)' = \frac{u'}{|u|\sqrt{u^2-1}} \; ; \; \left(\operatorname{arccsc} u\right)' = \frac{-u'}{|u|\sqrt{u^2-1}}$$

E.g. $y = \arccos(x^4)$. Find $\frac{dy}{dx}$

$$\frac{dy}{dx} = -\frac{4x^3}{\sqrt{1 - x^8}}$$

E.g. $g(x) = 5\arccos\left(\overset{u}{\boxed{\frac{x}{2}}}\right)$. Find $g'(x)$

$$g'(x) = 5 \cdot \frac{-\frac{1}{2}}{\sqrt{1 - \frac{x^2}{4}}} = \frac{-\frac{5}{2}}{\sqrt{\frac{4 - x^2}{4}}}$$

$$= \frac{-\frac{5}{2}}{\frac{\sqrt{4 - x^2}}{2}} = -\frac{5}{2} \cdot \frac{2}{\sqrt{4 - x^2}}$$

$$= \frac{-\cancel{10}^{\,5}}{\cancel{2}\sqrt{4 - x^2}} = \boxed{-\frac{5}{\sqrt{4 - x^2}}}$$

E.g: $h(x) = x^2 \cdot \arctan(7x)$

Product Rule: $h'(x) = 2x \cdot \arctan(7x) + x^2 \cdot \frac{7}{1 + 49x^2}$

$$h'(x) = \boxed{2x\arctan(7x) + \frac{7x^2}{1 + 49x^2}}$$

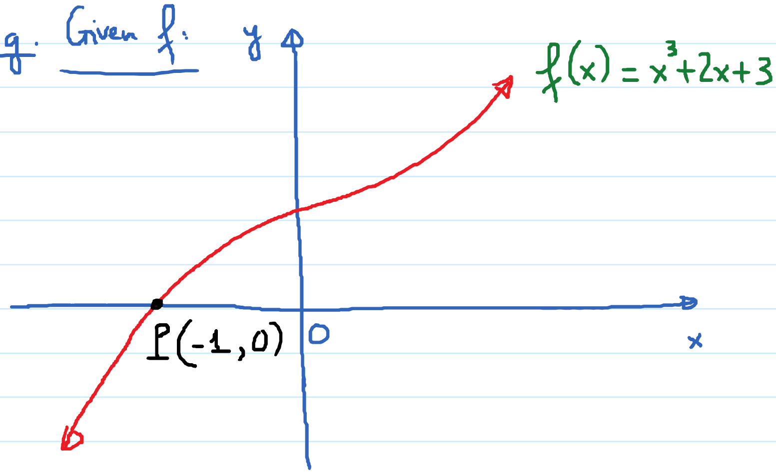
E.g. $y = \left(1 + \operatorname{arccot}(x)\right)^7$

$$\frac{dy}{dx} = \underbrace{7\left(1 + \operatorname{arccot}(x)\right)^6}_{d \text{ of outside}} \cdot \underbrace{\left(\frac{-1}{1+x^2}\right)}_{d \text{ of inside}}$$

$$= \boxed{\frac{-7\left(1 + \operatorname{arccot}(x)\right)^6}{1+x^2}}$$

What else is the I.F.T used for?

E.g. Given f:



Q1: Find the slope of the tangent line to the inverse function of f; i.e., f^{-1} at $Q(0,-1)$.

Q2: Find the equation of the tangent line to the graph of f^{-1} at $Q\,(0,-1)$.

Sol:

(a) Slope of tangent line to f^{-1} at $(0,-1) \equiv \left(f^{-1}\right)'(0)$

By I.F.T.

$$\left(f^{-1}\right)'(0) = \frac{1}{f'(\underbrace{f^{-1}(0)}_{-1})} = \frac{1}{\boxed{f'(-1)}}$$

We have $f'(x) = 3x^2+2$.

So, $f'(-1) = \boxed{5}$

Hence, $\left(f^{-1}\right)'(0) = \frac{1}{5}$.

$\therefore$ Slope of tangent line to f^{-1} at $Q(0,-1)$ is $\boxed{\frac{1}{5}}$

(b) Equation of tangent line to f^{-1} at Q.

$$\boxed{y = \frac{1}{5}x - 1}$$