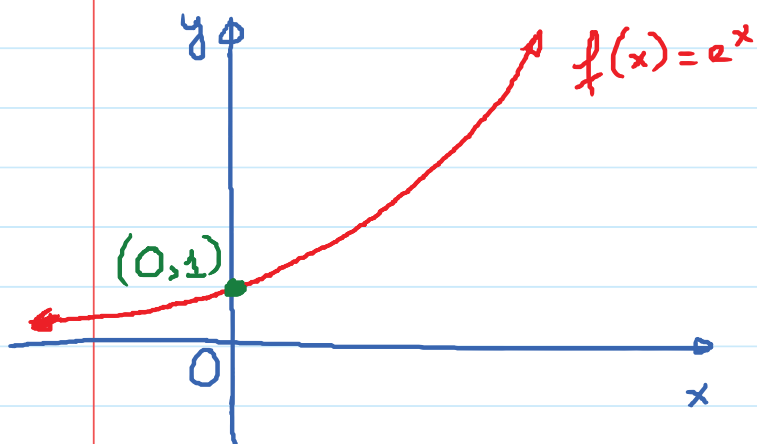
3.9. Derivatives of Log and Exponential Functions

Monday, February 25, 2019 9:09 AM

① Derivative of Exponential Functions with base e.

Derivative of $\boxed{f(x) = e^x}$

Base e : e is a constant, $e \simeq 2.71828\ldots$



Domain = $(-\infty, \infty)$

Range = $(0, \infty)$

If $f(x) = e^x$, then $f'(x) = e^x$

In Newton's notation: $\boxed{[e^x]' = e^x}$

In Leibnitz's notation : $\boxed{\frac{d}{dx}[e^x] = e^x}$

Mixing this with chain rule: u is a function of x

In leibnitz notation: $\boxed{\frac{d}{dx}[e^u] = e^u \cdot \frac{du}{dx}}$

In Newton notation: $\boxed{\left[e^{f(x)}\right]' = e^{f(x)} \cdot f'(x)}$

Ex. Find the derivative

(a) $\dfrac{d}{dx}\left[e^{2019}\right] = 0$ (b) $\dfrac{d}{dx}\left[e^{-x}\right]$

(c) $\dfrac{d}{dx}\left[e^{kx}\right]$, k is a constant.

(d) $\dfrac{d}{dx}\left[e^{\sec x}\right]$ (e) $\dfrac{d}{dx}\left[\arctan\left(e^{x}\right)\right]$

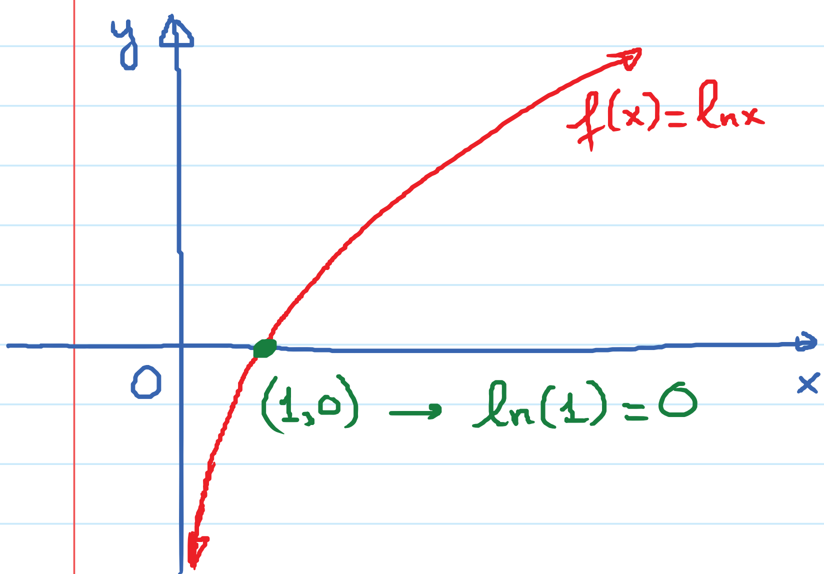
(f) $\dfrac{d}{dx}\left[\dfrac{x}{e^{2x}}\right]$

Done in class.

(2) Derivative of the natural log function.

The natural log function is the inverse function of the exponential function of base e.

$$f(x) = \ln(x)$$



Domain $= (0, \infty)$

Range $= (-\infty, \infty)$

If $f(x) = \ln x$, then $f'(x) = \frac{1}{x}$

In Newton notation: $\boxed{[\ln x]' = \frac{1}{x}}$

In Leibnitz notation: $\boxed{\frac{d}{dx}[\ln x] = \frac{1}{x}}$

Mixing this with chain rule: u: a function of x

In Leibnitz: $\boxed{\frac{d}{dx}[\ln u] = \frac{1}{u} \cdot \frac{du}{dx}}$

In Newton: $\boxed{[\ln u]' = \frac{u'}{u}}$

Ex. Find the derivative

(a) $\frac{d}{dx}\left[\ln(\cos x)\right]$ (b) $\frac{d}{dx}\left[\ln(\sin x)\right]$

(c) $\frac{d}{dx}\left[\ln\left(1+\frac{6}{x}\right)\right]$ (d) $\frac{d}{dx}\left[\ln\left(x+\sqrt{3+x^2}\right)\right]$

Ans:

(a) $-\tan x$ (b) $\cot x$.

(c) $-\frac{6}{x(x+6)}$ (d) $\frac{1}{\sqrt{x^2+3}}$

(3) Derivative of exp. and log. functions with base other than e.

Let a be a number with $a > 0$ and $a \neq 1$.

If $f(x) = a^x$, then $f'(x) = a^x \cdot \ln a$

In Leibnitz: $\boxed{\frac{d}{dx}\left[a^x\right] = a^x \cdot \ln a}$

In Newton: $\boxed{\left[a^x\right]' = a^x \ln a}$

Mixing this with chain rule: u is a function of x.

$$\boxed{\frac{d}{dx}\left[a^u\right] = a^u \cdot \ln a \cdot \frac{du}{dx}}$$

or $\boxed{\left[a^u\right]' = a^u \cdot \ln a \cdot u'}$

If $f(x) = \log_a x$, then $f'(x) = \dfrac{1}{x \ln a}$.

In Leibnitz: $\boxed{\dfrac{d}{dx}\left[\log_a x\right] = \dfrac{1}{x \ln a}}$

In Newton: $\boxed{\left[\log_a x\right]' = \dfrac{1}{x \ln a}}$

With Chain Rule: u is a function of x:

$$\boxed{\frac{d}{dx}\left[\log_a u\right] = \frac{1}{u \ln a} \cdot \frac{du}{dx}}$$ or

$$\boxed{\left[\log_a u\right]' = \frac{u'}{u \ln a}}$$