E.g. $\frac{d}{dx}\left[2^x\right] = 2^x \cdot \ln(2)$

$\frac{d}{dx}\left[\log_{2019}(x)\right] = \frac{1}{x \ln(2019)}$

$\frac{d}{dx}\left[3^{\tan x}\right] = 3^{\tan x} \cdot \ln(3) \cdot \sec^2 x.$

$\frac{d}{dx}\left[\log_7(x^2+x+1)\right] = \frac{2x+1}{(x^2+x+1)\cdot \ln(7)}$

4) Method of Logarithmic Differentiation

Useful Properties of Log function.

(a) $\ln[uv] = \ln(u) + \ln(v)$ (Product rule for log.)

(b) $\ln\left[\frac{u}{v}\right] = \ln(u) - \ln(v)$ (Quotient rule for log.)

(c) $\ln[u^p] = p\ln(u)$ (Power rule for log)

E.g. to illustrate how to use log. differentiation to find derivatives.

Given $y = x^x$. Find $\frac{dy}{dx}$.

Note: $\cancel{\frac{d}{dx}[x^x] = x^x \cdot \ln x}$ (b/c base is NOT a constant)

$\cancel{\frac{d}{dx}[x^x] = x \cdot x^{x-1}}$ (b/c power is NOT a constant)

Correct way to do this.

$\ln(y) = \ln(x^{\boxed{x}})$ (Take ln of both sides)

$\ln(y) = x\ln(x)$ (Apply the power rule for ln)

Then we differentiate both sides (take $\frac{d}{dx}$ of both sides)

$$\frac{d}{dx}[\ln(y)] = \frac{d}{dx}[x\ln(x)]$$

chain rule and rule for deri. of ln

product rule

$$\frac{1}{y} \cdot \frac{dy}{dx} = 1 \cdot \ln(x) + x \cdot \frac{1}{x}$$

algebra.

$$\frac{1}{y} \cdot \boxed{\frac{dy}{dx}} = \ln(x) + 1$$

$$\frac{dy}{dx} = [\ln(x) + 1] \cdot \boxed{y}$$

(Multiply both sides by y to get $\frac{dy}{dx}$ by itself)

$$\boxed{\frac{dy}{dx} = [\ln(x) + 1] \cdot x^x}$$

(Replace y by its formula)

E.g. HW: $y = \left(\sin(4x)\right)^{3x}$. Find $\frac{dy}{dx}$.

Done in class.

Note: Log. Differentiation can be applied to find the derivative of functions of form:

$$y = \left[f(x)\right]^{g(x)}$$

(Have variables in both the base and exp)

It can also be applied in other situations:

E.g. $y = \left(\frac{1+2x^2}{7-8x}\right)^4$. Find $\frac{dy}{dx}$

Note: We could use power rule, chain rule, quotient rule.

Let's try log. diff.

$$\ln y = \ln\left(\frac{1+2x^2}{7-8x}\right)^{\boxed{4}}$$

$$\ln y = 4\cdot\left[\ln(1+2x^2) - \ln(7-8x)\right]$$

(properties of ln to expand RHS)

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Take derivative:

$$\frac{1}{y} \cdot \frac{dy}{dx} = 4 \cdot \left[\frac{4x}{1+2x^2} - \frac{-8}{7-8x} \right]$$

$$\boxed{\frac{dy}{dx} = 4 \cdot \left[\frac{4x}{1+2x^2} + \frac{8}{7-8x} \right] \cdot \left(\frac{1+2x^2}{7-8x} \right)^4}$$