$f(x) = e^x$	$\int e^x\,dx = e^x + C$
$f(x) = \dfrac{1}{1+x^2}$	$\int \dfrac{1}{1+x^2}\,dx = \arctan(x) + C$
$f(x) = \dfrac{1}{\sqrt{1-x^2}}$	$\int \dfrac{1}{\sqrt{1-x^2}}\,dx = \arcsin(x) + C$
$f(x) = k$ (k: constant)	$\int k\,dx = kx + C$

$$\int \tan x\,dx = \ln(\sec(x)) + C$$

Check: $$\left[\ln(\sec(x))\right]' = \frac{1}{\sec(x)} \cdot \sec(x) \cdot \tan(x) = \tan(x)$$

$$\int \sec(x)\,dx = \ln(\sec(x) + \tan(x)) + C$$

Check: $$\left[\ln(\sec(x) + \tan(x))\right]' = \frac{\sec(x)\tan(x) + \sec^2(x)}{\sec(x) + \tan(x)} = \sec(x)$$

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Useful properties for Antiderivatives:

E.g. $\int \cos(x)\,dx = \sin(x) + C.$

$$\int 2019\cdot\cos(x)\,dx \overset{?}{=} 2019\cdot\sin(x) + C$$

In general,

$$\boxed{\int k\cdot f(x)\,dx = k\cdot\int f(x)\,dx}$$

k is any constant

In words, the antiderivative of a constant times a function is the same as that constant times the antiderivative of the function.

E.g. $\int \sec^2(x)\,dx = \tan(x) + C$; $\int x^3\,dx = \frac{x^4}{4} + C$

$$\int \left(\sec^2(x) + x^3\right)dx = \tan(x) + \frac{x^4}{4} + C$$

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$$\int (f(x) \pm g(x))\,dx = \int f(x)\,dx \pm \int g(x)\,dx$$

Note: $\int \sec^2(x)\,dx = \tan(x) + C$; $\int x^3\,dx = \frac{x^4}{4} + C$

~~$\int x^3 \cdot \sec^2(x)\,dx = \frac{x^4}{4} \cdot \tan(x) + C$~~

~~$\int f(x) \cdot g(x)\,dx = \int f(x)\,dx \cdot \int g(x)\,dx$~~ → wrong.

More E.g. $\int \sqrt[5]{x^3}\,dx = \int x^{\frac{3}{5}}\,dx = \frac{x^{\frac{3}{5}+1}}{\frac{3}{5}+1} + C$

(Rewrite; $\frac{3}{5}$ = n; Anti-Power rule)

$$= \frac{x^{\frac{8}{5}}}{\frac{8}{5}} + C = \boxed{\frac{5}{8} x^{\frac{8}{5}} + C}$$

E.g. $\int \frac{1}{x^5}\,dx = \int x^{-5}\,dx = \frac{x^{-5+1}}{-5+1} + C = \frac{x^{-4}}{-4} + C$

$$= \boxed{-\frac{1}{4x^4} + C}$$

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E.g. $\int \frac{x+8}{\sqrt{x}}\,dx = \int (x+8)\cdot x^{-\frac{1}{2}}\,dx$

Rewrite

distribute

$= \int \left(x^{\frac{1}{2}}\cdot x^{-\frac{1}{2}} + 8\cdot x^{-\frac{1}{2}}\right) dx$

Simplify

$= \int \left(x^{\frac{1}{2}} + 8\cdot x^{-\frac{1}{2}}\right) dx$

Split

$= \int x^{\frac{1}{2}}\,dx + 8\cdot\int x^{-\frac{1}{2}}\,dx$

Anti power rule

$= \frac{x^{\frac{3}{2}}}{\frac{3}{2}} + 8\cdot\frac{x^{\frac{1}{2}}}{\frac{1}{2}} + C$

$= \frac{2}{3}x^{\frac{3}{2}} + 16\cdot x^{\frac{1}{2}} + C$