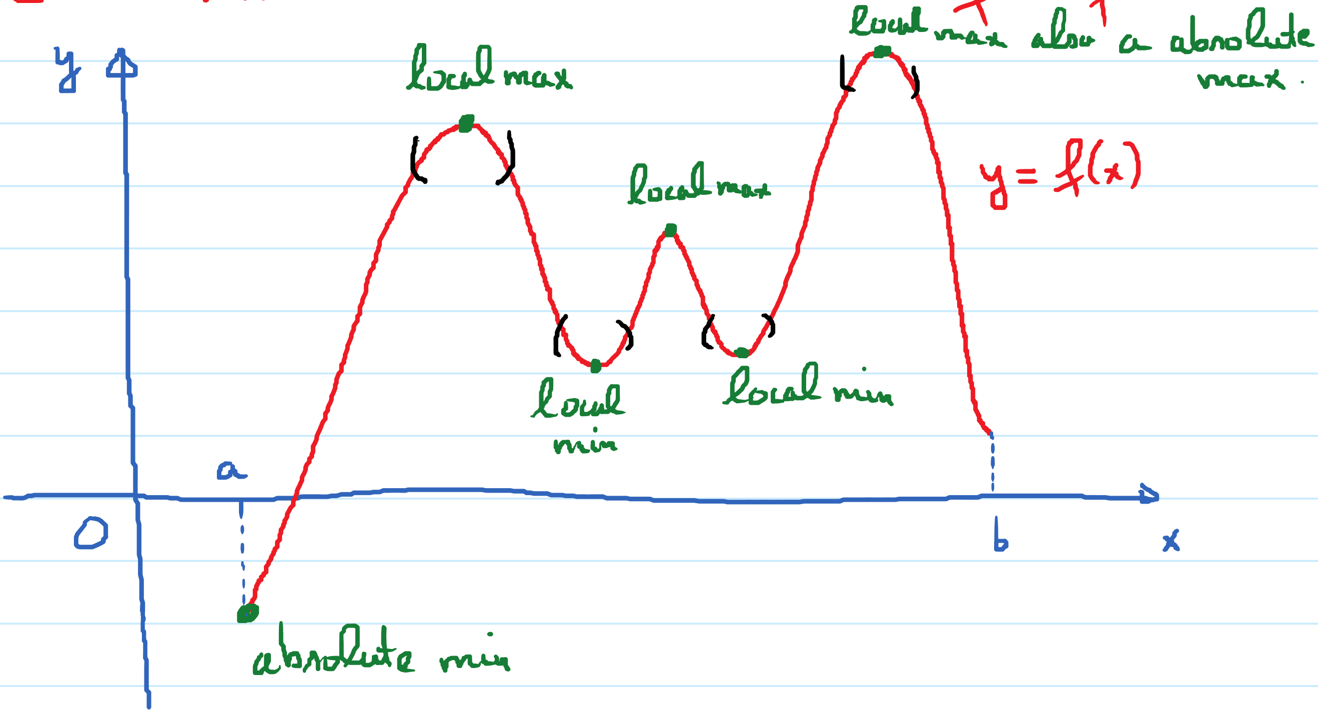
4.3. Find Maxima and Minima of a function

Monday, March 18, 2019 8:14 AM

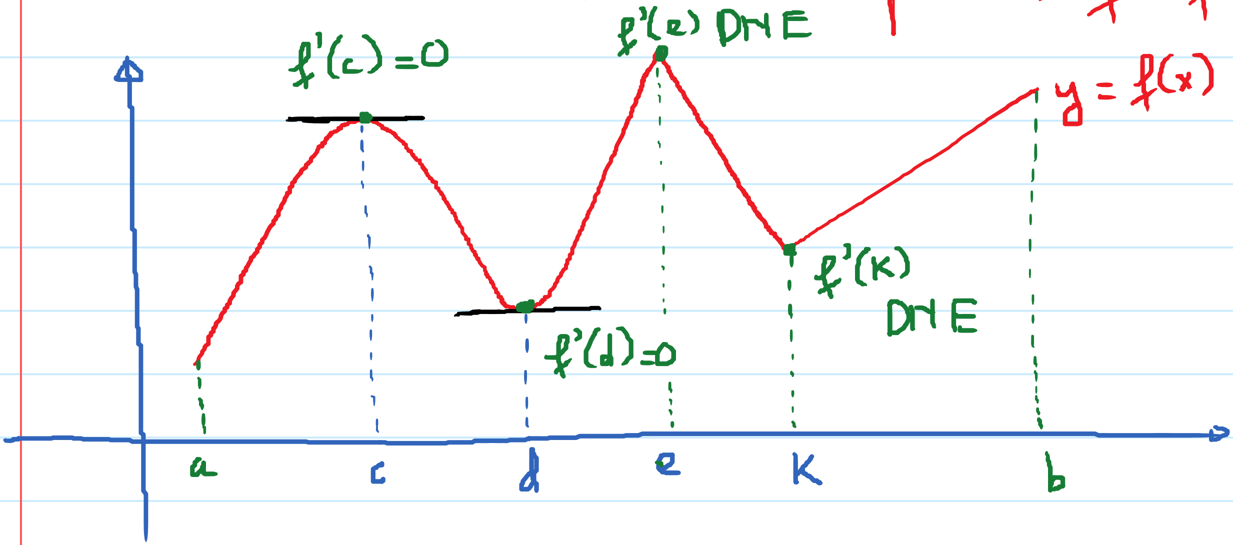
<u>Objectives:</u> (1) Find critical number(s) (critical points) of a function

(2) Apply the closed interval method to find maxima and minima of a function f over a closed interval $[a, b]$

Local Extrema vs. Absolute Extrema of a function.



Critical number(s) (or critical points) of a function.



Def: Let c be a number in the domain of a function $y=f(x)$. We say that c is a critical number of f if either $\boxed{(1)\ f'(c)=0}$ or $\boxed{(2)\ f'(c)\text{ DNE.}}$

E.g. $f(x)=(x+2)^3$. **Domain:** $(-\infty,\infty)$

$f'(x)=3(x+2)^2$

$f'(-2)=0$ and -2 is in Domain.

So, $x=-2$ is a critical number of f.

E.g. $g(x)=\sqrt{x}$ **Domain:** $[0,\infty)$

$g'(x)=\dfrac{1}{2\sqrt{x}}$. $g'(0)$ is undefined (DNE) and 0 is in Domain. So, $x=0$ is a critical # of g.

To find the critical #(s) of a function f

Step 1: Find the domain of f.

Step 2: Find the derivative f'.

Step 3: Set $f' = 0$ and solve for x.

and find values of x at which f' is undefined.

Step 4: The values of x in Step 3 that are in the domain will be the critical numbers of f.

Ex. Find the critical #(s) of the given function.

ⓐ $f(x) = x^3 - 6x^2 + 9x + 1$.

ⓑ $g(x) = \ln(1 - x)$

ⓒ $h(x) = \dfrac{4x}{x^2 + 1}$

ⓓ $j(x) = x^{\frac{3}{5}}(4 - x)$

Sol: ⓐ • Domain: $(-\infty, \infty)$

• $f'(x) = 3x^2 - 12x + 9$

• $3x^2 - 12x + 9 = 0 \longrightarrow 3(x^2 - 4x + 3) = 0$

$3(x-1)(x-3)=0 \rightarrow x-1=0$ or $x-3$

$\rightarrow x=1$ or $x=3$.

No values of x for which f' is undefined.

<u>Conclusion:</u> critical numbers are 1 and 3.

(b) $g(x) = \ln(\boxed{1-x})$

* To find domain: $1-x>0 \rightarrow x<1$

Domain: $(-\infty, 1)$

* Find f'. (Formula $(\ln u)' = \dfrac{u'}{u}$)

$f'(x) = (\ln(1-x))' = \dfrac{-1}{\boxed{1-x}}$.

* Set $f'=0$: $\dfrac{-1}{1-x} = 0 \longrightarrow$ No solution (s)

Find x s.t. f' is undefined : denom $=0 \rightarrow x=1$.

* Conclusion: Since 1 is not in domain, f has <u>NO</u> critical number.

(c) $h(x) = \dfrac{4x}{x^2+1}$

* Find domain: Domain $= (-\infty, \infty)$ (Denom. can never be zero)

quotient rule

* $h'(x) = \dfrac{4\cdot(x^2+1) - 4x\cdot 2x}{(x^2+1)^2}$

$$h'(x) = \frac{4x^2+4-8x^2}{(x^2+1)^2} = \frac{4-4x^2}{(x^2+1)^2}.$$

* Set $h' = 0$: $\dfrac{4-4x^2}{(x^2+1)^2} = 0 \rightarrow 4-4x^2 = 0$

$\rightarrow x = \pm 1$ (in domain)

h' is undefined? No values of x b/c denom. cannot be 0.

* Conclusion: critical numbers: 1 and -1.

(d) $j(x) = x^{\frac{3}{5}}\cdot(4-x)$. Domain: $(-\infty, \infty)$

$$j'(x) = \frac{3}{5}x^{-\frac{2}{5}}\cdot(4-x) + x^{\frac{3}{5}}\cdot(-1)$$

$$= \frac{12}{5}x^{-\frac{2}{5}} \underbrace{-\frac{3}{5}x^{\frac{3}{5}} - x^{\frac{3}{5}}}$$

$$= \frac{12}{5} x^{\boxed{-\frac{2}{5}}} - \frac{8}{5} x^{\frac{3}{5}}$$

$$= \frac{12}{5x^{2/5}} - \frac{8x^{3/5}}{5} \cdot \frac{x^{2/5}}{x^{2/5}}$$

$$j'(x) = \frac{12 - 8x}{5x^{2/5}}$$

$j'(x) = 0$ when $12 - 8x = 0 \to \boxed{x = \frac{3}{2}.}$

$j'(x)$ is undefined when $5x^{2/5} = 0 \to \boxed{x = 0}.$ are in Domain

Conclusion: critical #s are $\frac{3}{2}$ and 0.

Closed interval method to find absolute maximum and absolute minimum of a function f on $[a,b]$.

E.g. $f(x) = x^3 - 6x^2 + 9x + 1$.

Find the absolute minimum and absolute maximum of this function on $[-1, 2]$

Closed Interval Method: