4.5 How does the derivative affect the shape of a graph

Wednesday, March 20, 2019 8:05 AM

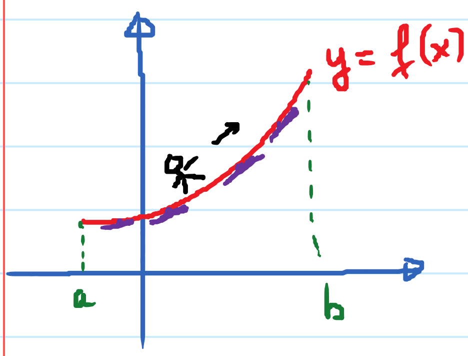
Obj: (1) Intervals of increasing / decreasing

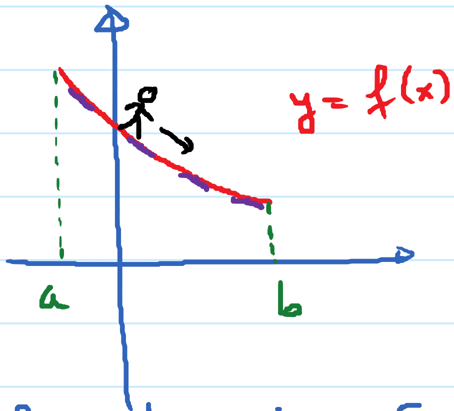
(2) First Derivative Test to find local (relative) extrema

(3) Intervals of concavity.

(4) Second Derivative Test to find local extrema.

* What does f' tell us about f ?





f is increasing on $[a,b]$ f is decreasing on $[a,b]$

(I) Theorem:

(a) If $\boxed{f'(x) > 0}$ for every x in an interval I ($f' > 0$ on I), then $\boxed{f}$ is $\boxed{\text{increasing}}$ on I

(b) If $\boxed{f'(x) < 0}$ for every x in an interval I ($f' < 0$ on I), then $\boxed{f}$ is $\boxed{\text{decreasing}}$ on I.

$$\boxed{12x^3 - 12x^2 - 24x} < 0$$
$$> 0$$

E.g. $f(x) = 3x^4 - 4x^3 - 12x^2 + 5$. Domain $(-\infty, \infty)$

Q: Determine the interval(s) on which f is increasing/decreasing?

Process: ① Find f'

② Set $f' = 0$ or f' is undefined

(find critical numbers)

③ Make a number line and use test points to determine when $f' < 0$ or $f' > 0$.

Sol: ① $f'(x) = 12x^3 - 12x^2 - 24x$.

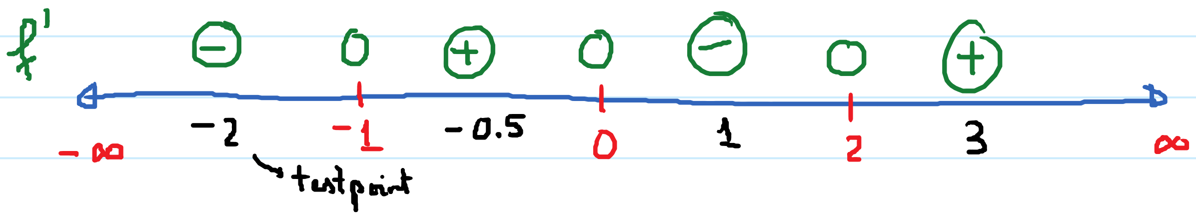
② $f'(x) = 0 \longrightarrow 12x^3 - 12x^2 - 24x = 0$

$\longrightarrow 12x(x^2 - x - 2) = 0$

$\longrightarrow 12x(x-2)(x+1) = 0$

$\longrightarrow \boxed{x = 0}$; $\boxed{x = 2}$; $\boxed{x = -1}$ critical numbers.

③ Draw a number line:



Conclusion:

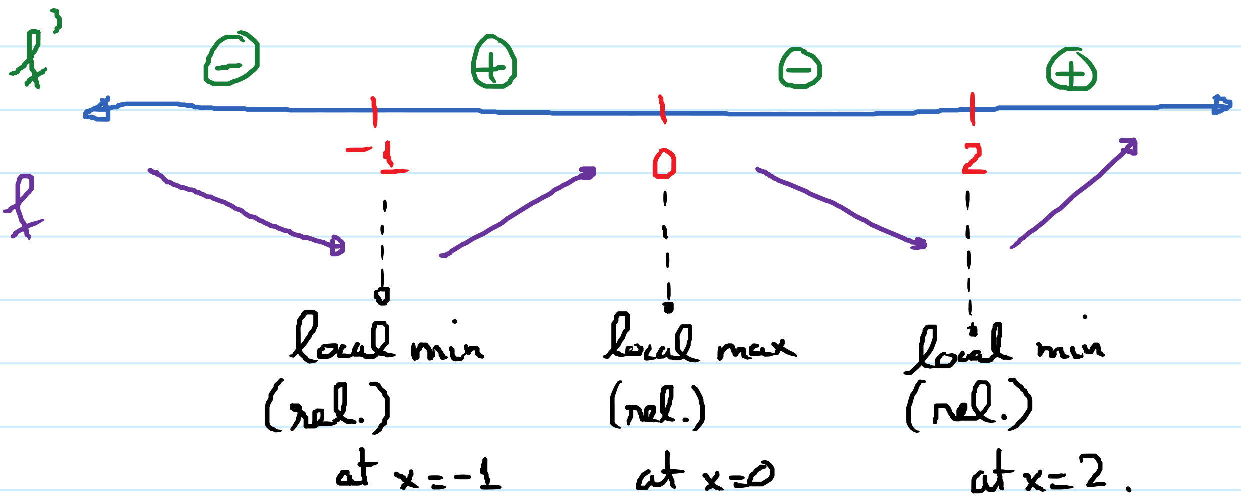
The function f is increasing on:

$$\boxed{(-1,0) \cup (2,\infty)}$$

The function f is decreasing on:

$$\boxed{(-\infty,-1) \cup (0,2)}$$

Moreover,



Find values of function at local max/min → plug x values to $f(x)$

min value

At $x=-1$; $\boxed{f(-1)} = \dots$

max value

At $x=0$; $\boxed{f(0)} = 5 \dots$

At $x=2$; $\boxed{f(2)} = \dots$ → min value.

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Summary of the first derivative test

→ intervals of increasing / decreasing

→ local max / min

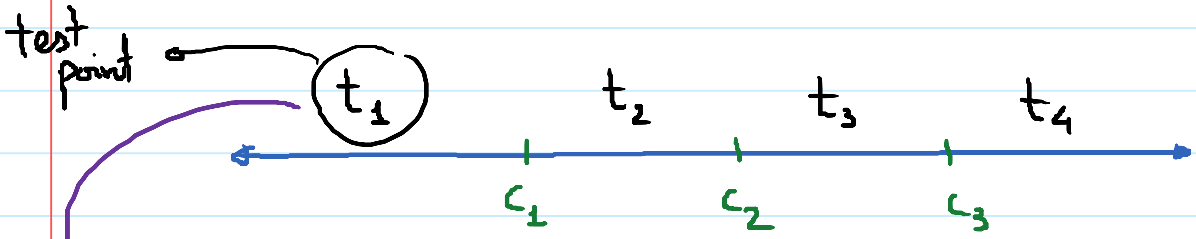
Step 1: Find the 1^{st} derivative f'

Step 2: Find the critical numbers

→ $f' = 0$ → solve for x

→ f' undefined → find x.

Step 3: Draw a number line. ($c_1, c_2, c_3 \ldots$ critical #s)



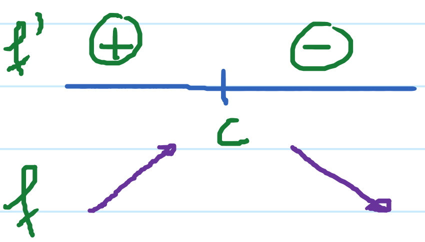
plug test points into the derivative f' to determine the sign(s) of f' on the interval(s) in between the critical number(s)

$(x-2)^2(x-3)$

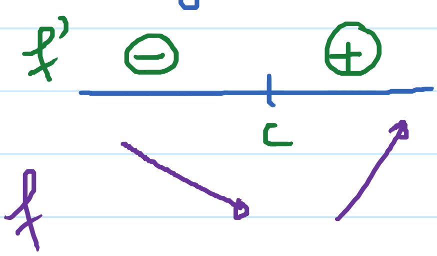
Step 4: $f' > 0 \rightarrow f$ increasing

$f' < 0 \rightarrow f$ decreasing

Step 5:



(rel) local max at $x=c$



local min at $x=c$