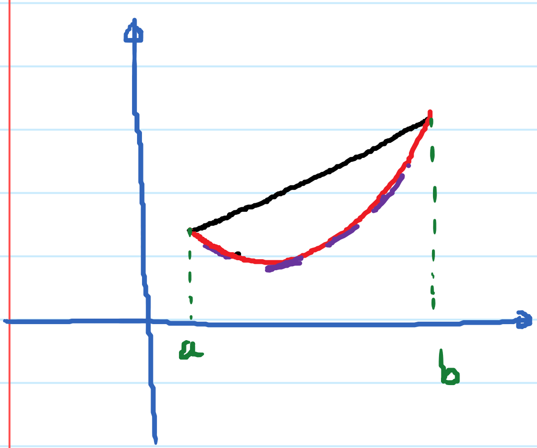
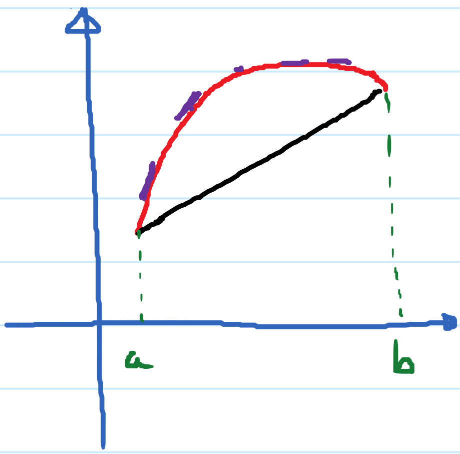
Note: If f' does not change sign when we pass through a critical number $x=c$ then we have neither a max nor a min at $x=c$.

(2) What does f'' tell us about f?



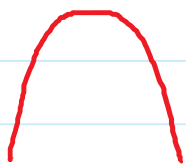
f is concave up on $[a,b]$



f is concave down on $[a,b]$

(II) Theorem:

(1) If $\boxed{f''(x)>0}$ for every x in an interval I ($f''>0$ on I), then $\boxed{f}$ is $\boxed{\text{concave up}}$ on I.



(2) If $\boxed{f''(x)<0}$ for every x in an interval I, ($f''<0$ on I), then $\boxed{f}$ is $\boxed{\text{concave down}}$ on I.

E.g. $f(x) = x^4 - 5x^3$.

Intervals on which f is concave up / concave down.

Step 1: Find f''

$$f'(x) = 4x^3 - 15x^2$$

$$f''(x) = 12x^2 - 30x$$

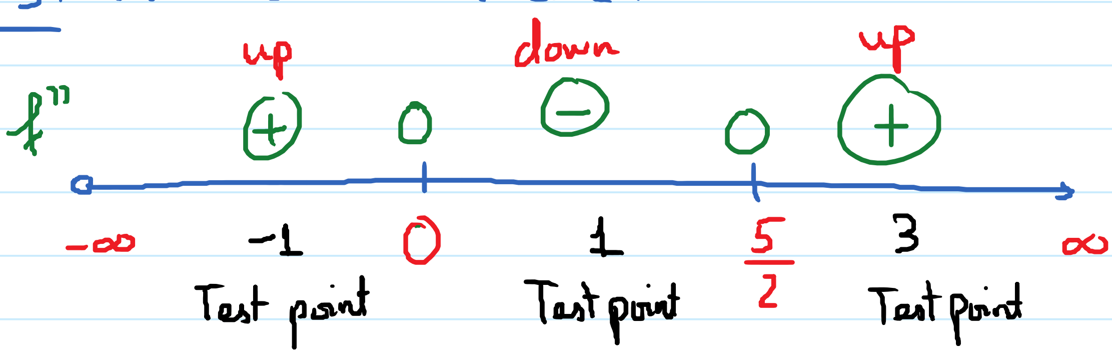
Step 2:
- Set $f'' = 0$ and solve for x
- f'' is undefined and find x

$$f''(x) = 0 \longrightarrow 12x^2 - 30x = 0$$

$$6x(2x-5) = 0$$

$$x = 0 \; ; \; x = \frac{5}{2}.$$

Step 3: Draw a number line.



Conclusion: f is concave up on $(-\infty, 0) \cup \left(\frac{5}{2}, \infty\right)$

f is concave down on $\left(0, \frac{5}{2}\right)$

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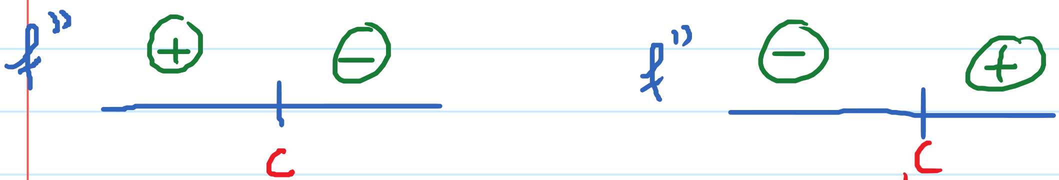
Definition: Inflection point

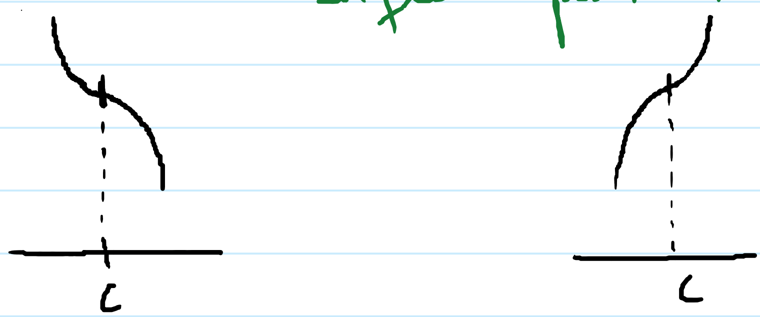
An inflection point is the point where the graph changes its concavity (from up to down or from down to up) as x changes across that point.

In terms of f'': If $f''(c)=0$ or $f''(c)$ is undefined and f'' changes its sign across $x=c$ then at $x=c$, f has an inflection point.



$f''(c)=0$ or $f''(c)$ undefined

Inflection point at $x=c$.



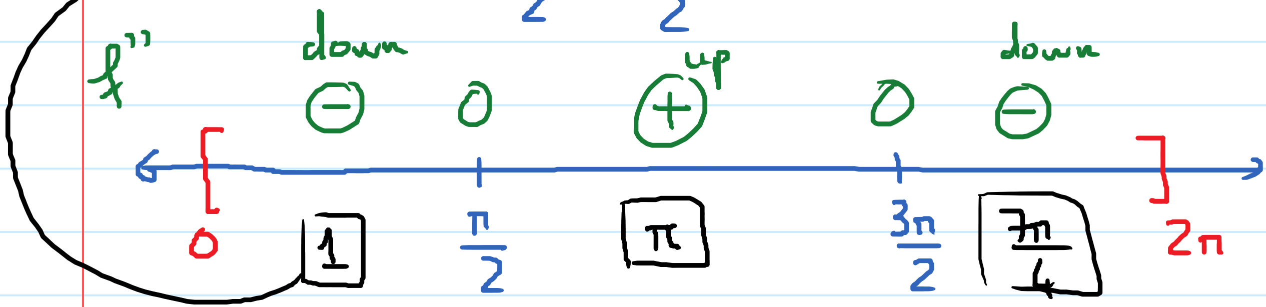
E.g. $f(x) = x + 7\cos x$ on $[0, 2\pi]$

Intervals of concavity? Inflection points.

$f'(x) = 1 - 7\sin x$

$f''(x) = \boxed{-7\cos x} = 0 \longrightarrow \cos x = 0$

$\longrightarrow x = \frac{\pi}{2}, \frac{3\pi}{2}$.

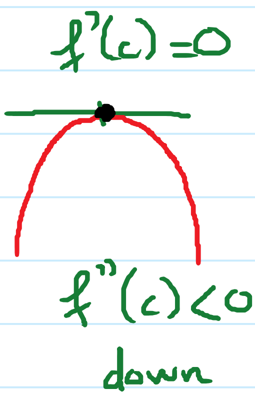


Concave down: $\left(0, \frac{\pi}{2}\right) \cup \left(\frac{3\pi}{2}, 2\pi\right)$

up: $\left(\frac{\pi}{2}, \frac{3\pi}{2}\right)$.

Inflection points: $\left(\frac{\pi}{2}, \frac{\pi}{2}\right)$; $\left(\frac{3\pi}{2}, \frac{3\pi}{2}\right)$

Note: The second derivative can also be used in many cases to determine local min/max.



* $f'(c) = 0$ and $f''(c) < 0$

Then f has a local max at $x = c$.

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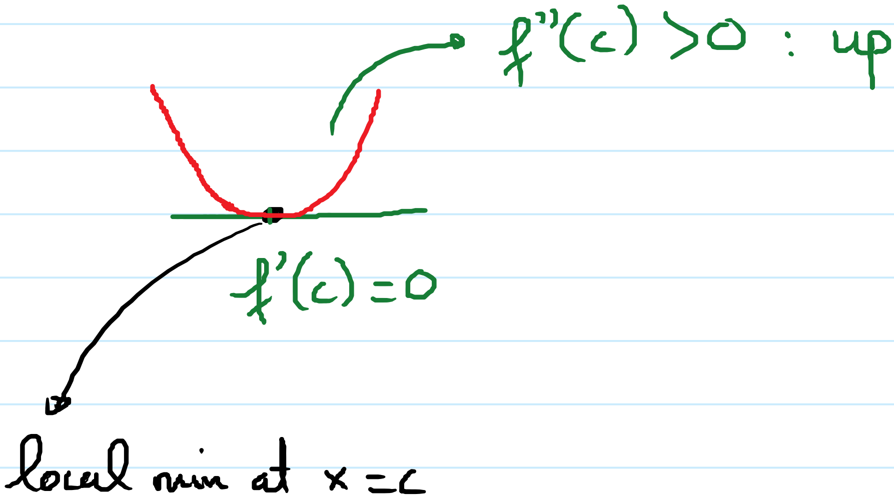
* $f'(c) = 0$ and $f''(c) > 0$

Then f has a local min at $x = c$.



* Note: $f''(c) = 0$; it gives no information about local extrema.