Wednesday, March 27, 2019 9:22 AM

E.g. $\lim_{x \to \infty} \boxed{x} \cdot \boxed{\tan\left(\boxed{\frac{1}{x}}\right)}$

$x \to \infty$, $\tan\left(\frac{1}{x}\right) \to 0$, $\frac{1}{x} \to 0$

Is this one of the indeterminate forms? If yes, which one?

Ans: Form $0 \cdot \infty$

$$\lim_{x \to \infty} x \cdot \tan\left(\frac{1}{x}\right) = \lim_{x \to \infty} \frac{\boxed{\tan\left(\frac{1}{x}\right)}}{\boxed{\frac{1}{x}}}$$

$\tan\left(\frac{1}{x}\right) \to 0$, $\frac{1}{x} \to 0$

So, we have a limit of the form $\frac{0}{0}$. Hence, we can apply L'Hopital.

$$= \lim_{x \to \infty} \frac{\sec^2\left(\frac{1}{x}\right) \cdot \cancel{\left(-\frac{1}{x^2}\right)}}{\cancel{-\frac{1}{x^2}}} = \lim_{x \to \infty} \sec^2\left(\boxed{\frac{1}{x}}\right)$$

$\frac{1}{x} \to 0$

$$= \sec^2(0) = \frac{1}{\cos^2(0)} = \boxed{1}.$$

E.g. $L = \lim_{x \to 0^+} (1+x)^{\frac{4}{x}}$

$0 \leftarrow$ $x > 0$

Form: 1^{∞}

Trick: Take ln of both sides:

$$\ln(L) = \lim_{x \to 0^+} \ln(1+x)^{\boxed{\frac{4}{x}}}$$

$$\ln(L) = \lim_{x \to 0^+} \frac{4}{x} \cdot \ln(1+x)$$

$$\ln(L) = \lim_{x \to 0^+} \frac{4\ln(1+x)}{x} \rightarrow \text{Form } \frac{0}{0}$$

$\downarrow$

L'Hopital

$$\ln(L) = \lim_{x \to 0^+} \frac{\frac{4}{1+x}}{1}$$

$$\underbrace{e^{\ln(L)}} = e^{4}$$

$$\longrightarrow L = e^{4}$$

So, $\boxed{\lim_{x \to 0^+} (1+x)^{\frac{4}{x}} = e^{4}}$

$\underline{\text{E.g.}}$ $L = \lim_{x \to 1^+} \left(\ln(x)\right)^{x-1}$ Form 0^0

$$\ln(L) = \lim_{x \to 1^+} \ln\left(\left(\ln(x)\right)^{x-1}\right)$$

$$= \lim_{x \to 1^+} (x-1) \cdot \ln(\ln(x)) \quad 0 \cdot (-\infty)$$

$$= \lim_{x \to 1^+} \frac{\ln(\ln(x))}{\frac{1}{x-1}} \quad \left(\frac{-\infty}{\infty}\right) \to \text{L'Hopital}$$

$$= \lim_{x \to 1^+} \frac{\frac{1}{\ln(x)} \cdot \frac{1}{x}}{-\frac{1}{(x-1)^2}}$$

$$= \lim_{x \to 1^+} \frac{1}{x\ln(x)} \cdot \left(-(x-1)^2\right)$$

$$= -\lim_{x \to 1^+} \frac{(x-1)^2}{x \cdot \ln(x)} \quad \left(\frac{0}{0}\right) \to \text{L'Hopital}$$

$$= -\lim_{x \to 1^+} \frac{2(x-1)}{\ln(x)+1}$$

$$\ln(L) = -\frac{0}{1} = 0 \longrightarrow L = e^0 = \boxed{1}.$$

E.g. $$\lim_{x\to 2^+}\left(\frac{8}{x^2-4}-\frac{x}{x-2}\right)$$

$$=\lim_{x\to 2^+}\frac{8}{(x-2)(x+2)}-\frac{x\cdot(x+2)}{(x-2)(x+2)}$$

$$=\lim_{x\to 2^+}\frac{8-x\cdot(x+2)}{(x-2)(x+2)}$$

$$=\lim_{x\to 2^+}\frac{8-x^2-2x}{x^2-4}\qquad\frac{0}{0}$$

$$=\lim_{x\to 2^+}\frac{-2x-2}{2x}=\frac{-6}{4}=\boxed{-\frac{3}{2}}$$