4.9. Newton's Method

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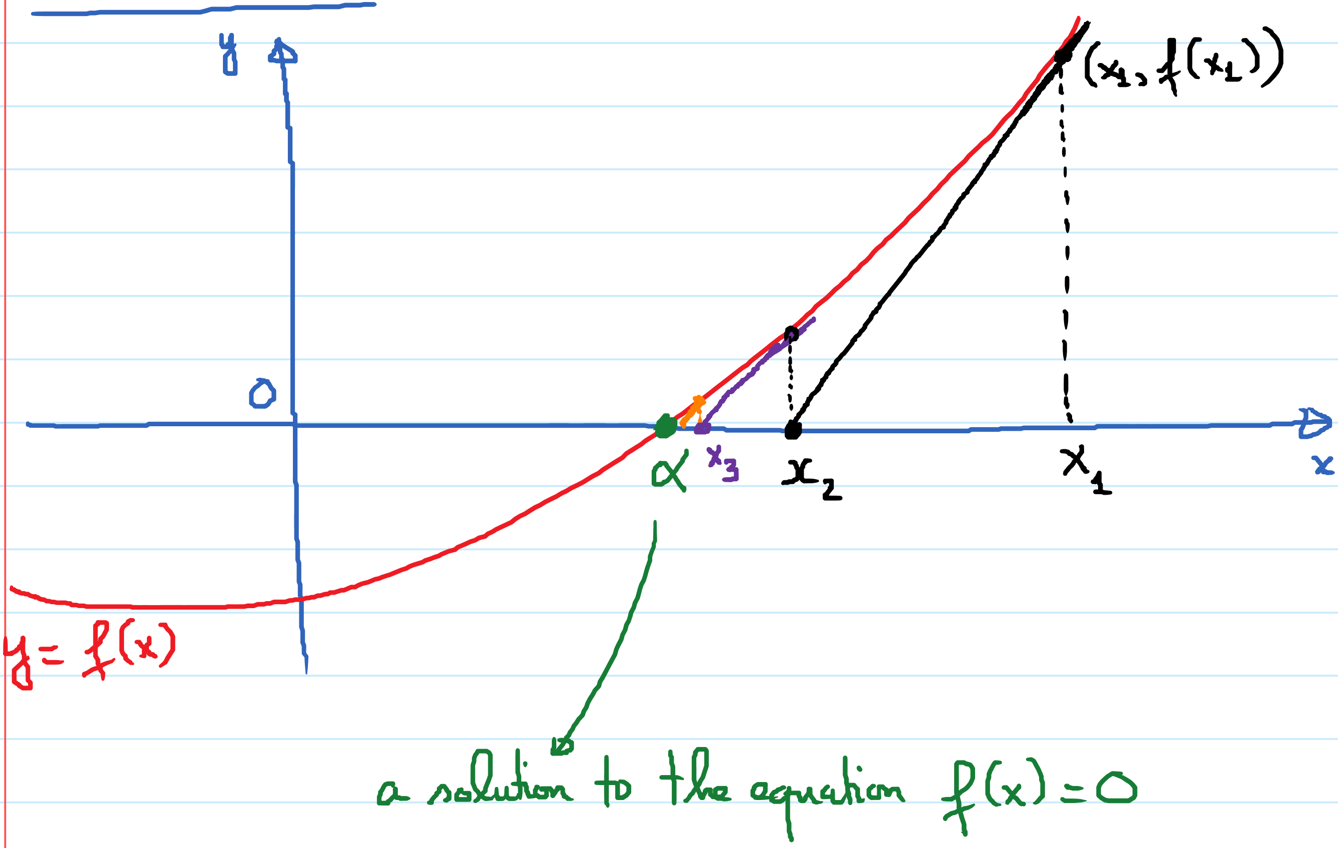
Goal: Use Newton's method to estimate the solutions to the equation $f(x)=0$.

Geometric idea:



Implementation of Newton's method

Step 1: Start with an initial guess $\boxed{x_1}$ → 1^{st} approx.

Step 2: The second approximation x_2 is the intersection between the tangent line to the graph of $y=f(x)$ at the point $(x_1, f(x_1))$ and the x-axis

Step 3: Repeat the process to obtain a sequence $x_3, x_4, x_5, \ldots$ of approximations to the solution of the equation.

If f is "nice", the sequence $x_3, x_4, x_5, \ldots$ will eventually gets close to the solution α of $f(x) = 0$.

→ Q: How do we find $x_2, x_3, x_4, \ldots$?

x_2 = intersection between the tangent line to graph of $y = f(x)$ at $(x_1, f(x_1))$ and x-axis.

* Tangent line to $y = f(x)$ at $\boxed{(x_1, f(x_1))}$ → point

Slope = $\boxed{f'(x_1)}$

Pt - Slope equation:

$$y - f(x_1) = f'(x_1) \cdot (x - x_1)$$

→ Equ. of tangent line at $(x_1, f(x_1))$

$$y = f'(x_1) \cdot (x - x_1) + f(x_1)$$

* x-intercept of this line.

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Set $y=0$ in the equation and solve for x

$$0 = f'(x_1)(x-x_1) + f(x_1)$$

$$\rightarrow \quad -f(x_1) = f'(x_1)(x-x_1)$$

$$\rightarrow \quad -\frac{f(x_1)}{f'(x_1)} = x - x_1$$

$$\rightarrow \quad \boxed{x} = x_1 - \frac{f(x_1)}{f'(x_1)}$$

x_2

So

$$\boxed{x_2 = x_1 - \frac{f(x_1)}{f'(x_1)}}$$

$$\boxed{x_3 = x_2 - \frac{f(x_2)}{f'(x_2)}}$$

In general, we have:

$$\boxed{x_{n+1} = x_n - \frac{f(x_n)}{f'(x_n)} \quad ; \quad n = 1, 2, 3, \ldots}$$

This is called Newton's method formula and it gives us the $(n+1)$-approximation from the n^{th} approx.