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E.g.

$$\int_0^{\pi/4} \frac{9 - 9\sin^2\theta}{3\cos^2\theta}\, d\theta = \int_0^{\pi/4} \frac{9(1-\sin^2\theta)}{3\cos^2\theta}\, d\theta$$

$$= \int_0^{\pi/4} \frac{9\cancel{\cos^2\theta}}{3\cancel{\cos^2\theta}}\, d\theta = 3\int_0^{\pi/4} 1\, d\theta = 3\cdot\theta \Big|_0^{\pi/4} = \boxed{\frac{3\pi}{4}}$$

E.g.

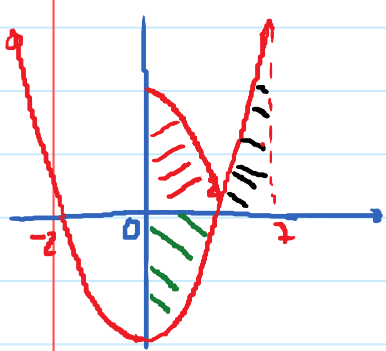
$$\int_0^1 \frac{x^2-5}{x^2+1}\, dx = \int_0^1 \frac{(x^2+1)-(6)}{x^2+1}\, dx$$

$$= \int_0^1 1\, dx - 6\int_0^1 \frac{1}{x^2+1}\, dx$$

$$= x\Big|_0^1 - 6\arctan(x)\Big|_0^1 = 1 - 6\frac{\pi}{4} = \boxed{1 - \frac{3\pi}{2}}$$

E.g.

$$\int_0^7 |x^2-4|\, dx = \int_0^2 |x^2-4|\, dx + \int_2^7 |x^2-4|\, dx$$




$$= \int_0^2 (-x^2+4)\, dx + \int_2^7 (x^2-4)\, dx$$

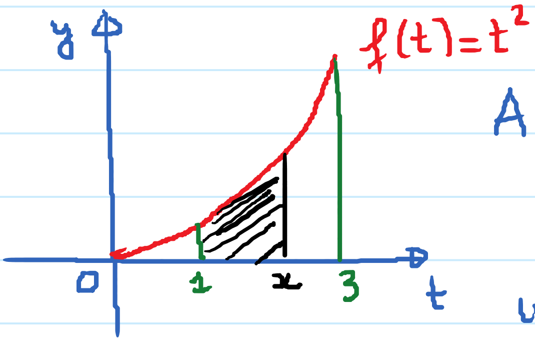
$$= \left(-\frac{x^3}{3}+4x\right)\Big|_0^2 + \left(\frac{x^3}{3}-4x\right)\Big|_2^7$$

$$= \boxed{97}$$

# Fundamental Theorem of Calculus, Part I

→ Lower bound and/or upper bound of the integral are variables.

<u>E.g.</u> $f(t) = \boxed{t^2}$ on $[1, 3]$




$A(x) = \int_1^x t^2 \, dt$ gives us the area under the graph of $f(t) = t^2$ on $[1, x]$

What is $A(x)$?

$$A(x) = \int_1^x t^2 \, dt = \frac{t^3}{3} \Big|_1^x = \frac{x^3}{3} - \frac{1}{3}$$

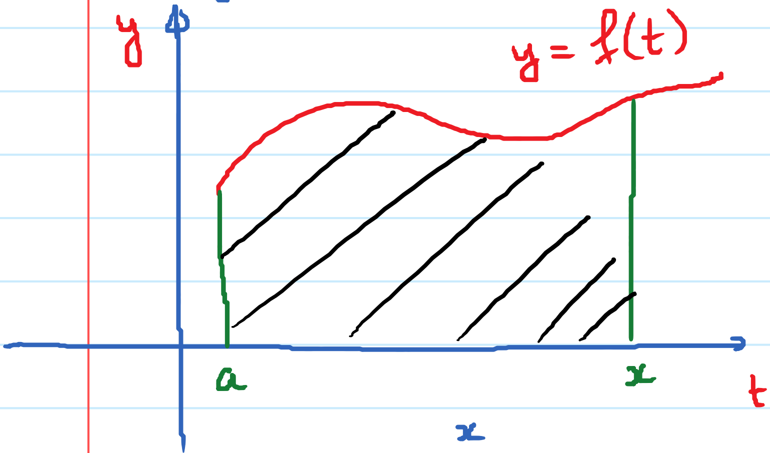
So, $A(x) = \frac{x^3}{3} - \frac{1}{3}$. $\left( \rightarrow A'(x) = \boxed{x^2} \right)$

Area from 1 to 3: $A(3) = \frac{(3)^3}{3} - \frac{1}{3} = \frac{26}{3}$

Area from 1 to 2: $A(2) = \frac{(2)^3}{3} - \frac{1}{3} = \frac{7}{3}$

→ $A(x)$ is a useful function.

In general,




The integral $\int_a^x f(t)\,dt$ is a function of $x$ and this integral gives us the area under graph of $y = f(t)$ from a to $x$.

Let $A(x) = \int_a^x f(t)\,dt$.

(Note: In many situations, we cannot find a "nice" formula for $A(x)$)

F.T.C. Part I allows us to find the derivative of $A(x)$.

F.T.C. Part I.

$$\boxed{\text{If } A(x) = \int_a^x f(t)\,dt, \text{ then } A'(x) = f(x)}$$

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A more compact form of F.T.C. part I

$$\underbrace{\frac{d}{dx}}\left(\underbrace{\int_a^x f(t)\,dt}\right) = \boxed{f(x)}$$

take derivative w.r.t. $x$ of the function in ( )

function of $x$

the integrand as a function of $x$.

E.g. $\cancel{\frac{d}{dx}}\left(\cancel{\int_0^{\boxed{x}}} e^{\boxed{t}^2}\,dt\right) = e^{x^2}$

$$\cancel{\frac{d}{dr}}\left(\cancel{\int_4^{\boxed{r}}} \frac{d\theta}{\sqrt{16-\boxed{\theta}^2}}\right) = \frac{1}{\sqrt{16-r^2}}$$

$$\frac{d}{dx}\left(\int_0^{\boxed{x^2}} \sec(\boxed{t})\,dt\right) = \sec(x^2)\cdot\boxed{(2x)}$$

derivative of $x^2$

General formula for F.T.C. part I

derivative of g

$$\frac{d}{dx}\left(\int_a^{\boxed{g(x)}} f(\boxed{t})\,dt\right) = f(g(x))\cdot g'(x)$$

derivative

E.g. $$\frac{d}{dx}\left(\int_1^{\boxed{x^4}} \frac{dt}{\boxed{t}^3+1}\right) = \frac{1}{(x^4)^3+1}\cdot(4x^3)$$

$$= \frac{4x^3}{x^{12}+1}.$$

E.g. $$\frac{d}{dx}\left(\int_{\sqrt{x}}^{x^2} \tan(8t)\,dt\right)$$

$$= \tan(8x^2)\cdot 2x - \tan(8\sqrt{x})\cdot\frac{1}{2\sqrt{x}}$$