FINAL EXAM REVIEW

Wednesday, May 1, 2019 8:16 AM

(1) $7xy^3 + 4x^3y = 1$. Find $\frac{dy}{dx}$.

$$\frac{d}{dx}\left(7xy^3\right) + \frac{d}{dx}\left(4x^3y\right) = \frac{d}{dx}(1) = 0$$

Product Rule

$$7\cdot\left[1\cdot y^3 + x\cdot 3y^2\cdot\frac{dy}{dx}\right] + 4\cdot\left[3x^2\cdot y + x^3\cdot\frac{dy}{dx}\right] = 0$$

$$7y^3 + \boxed{21xy^2\frac{dy}{dx}} + 12x^2y + \boxed{4x^3\frac{dy}{dx}} = 0$$

$$\frac{dy}{dx}\left[21xy^2 + 4x^3\right] = -7y^3 - 12x^2y$$

$$\boxed{\frac{dy}{dx} = \frac{-7y^3 - 12x^2y}{21xy^2 + 4x^3}} \longrightarrow \text{Answer.}$$

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(2) Step 1: Derivative.

$f(x) = 3x^{2/3} - 2x$. Domain: $(-\infty, \infty)$

$$f'(x) = \cancel{3}\cdot\frac{2}{\cancel{3}}x^{-1/3} - 2 = \frac{2}{x^{1/3}} - 2.$$

Step 2: Critical points:

* f' is undefined when $\boxed{x = 0}$ → 1^{st} critical point

* $f' = 0$: $\dfrac{2}{x^{1/3}} - 2 = 0 \rightarrow \dfrac{2}{x^{1/3}} = 2$ 2nd critical point

$\rightarrow \dfrac{1}{x^{1/3}} = 1 \rightarrow x^{1/3} = 1 \rightarrow \boxed{x = 1}$

Interval of consideration is $[-1, 1]$

Step 3: Plug in f

	x	$f(x) = 3x^{2/3} - 2x$
endpt	-1	5
critical	0	0
endpt and critical	1	1

Abs. max $= 5$; occurs at $x = -1$

Abs. min $= 0$; occurs at $x = 0$.

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(3) Step 1: 1st and 2nd Derivative

$$f(x) = 5x + \frac{8}{\sin x} = 5x + 8\csc x$$

$$f'(x) = 5 - 8\csc(x)\cdot\cot(x)$$

$$f''(x) = -8\left[-\csc(x)\cdot\cot(x)\cdot\cot(x) - \csc(x)\cdot\csc^2(x)\right]$$

$$= \boxed{8\left[\csc(x)\cdot\cot^2(x) + \csc^3(x)\right]}$$

Step 2: Sign Chart

$f''(x)$ is undefined when $\boxed{x=0}$

$f''(x)=0 \rightarrow 8\left[\csc(x)\cot^2(x)+\csc^3(x)\right]=0$

$\rightarrow \csc(x)\left[\cot^2(x)+\csc^2(x)\right]=0$

$\csc(x)=0$ or $\underbrace{\cot^2(x)}+\csc^2(x)=0$

$\dfrac{1}{\sin(x)}=0$ $\qquad$ $\csc^2(x)-1+\csc^2(x)=0$

$\rightarrow$ no solution $\qquad$ $2\csc^2(x)-1=0$

$\csc^2(x)=\dfrac{1}{2}$

$\sin^2(x)=2$

$\rightarrow$ no solution

$\rightarrow$ Sign chart $\qquad$ plug in f'' $\qquad$ plug in f''

Test pt $= -\frac{\pi}{2}$ $\qquad$ Test pt $= \frac{\pi}{2}$

x: $\quad (-\pi \quad\quad 0 \quad\quad \pi)$

f'': $\quad \ominus \quad$ undefined $\quad \oplus$

Conclusion: f is concave down on $(-\pi,0)$

f is concave up on $(0,\pi)$

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4 a $\lim_{x \to 0} \frac{7x + e^x - e^{8x}}{1 - \cos(3x)}$ $\left(\frac{0}{0}\right) \rightarrow$ L'Hopital

$= \lim_{x \to 0} \frac{7 + e^x - 8e^{8x}}{3\sin(3x)}$ $\left(\frac{0}{0}\right) \rightarrow$ L'Hopital

$= \lim_{x \to 0} \frac{e^x - 64e^{8x}}{9\cos(3x)}$

$= \frac{1 - 64}{9} = \boxed{-7}$

b $L = \lim_{x \to 1^+} (\ln(x))^{x-1}$ (0^0)

$\ln(L) = \lim_{x \to 1^+} (x-1) \cdot \ln(\ln(x))$ $(0 \cdot (-\infty))$

$= \lim_{x \to 1^+} \frac{\ln(\ln(x))}{\frac{1}{x-1}}$ $\left(\frac{-\infty}{\infty}\right) \rightarrow$ L'Hopital

$= \lim_{x \to 1^+} \frac{\frac{1}{\ln(x)} \cdot \frac{1}{x}}{-\frac{1}{(x-1)^2}} = \frac{\frac{1}{x \cdot \ln x}}{\frac{-1}{(x-1)^2}}$

$= -\lim_{x \to 1^+} \frac{(x-1)^2}{x \cdot \ln(x)}$ $\frac{0}{0}$

$$= -\lim_{x\to 1^+} \frac{2(x-1)}{1\cdot \ln x + x\cdot \frac{1}{x}}$$

$$= -\lim_{x\to 1^+} \frac{2(x-1)}{\ln x + 1} = -\frac{0}{1} = 0$$

So, $\ln L = 0$. So, $L = e^0 = \boxed{1}$

#5 a $\int \sqrt[5]{x^3}\,dx = \int x^{3/5}\,dx = \frac{x^{\frac{3}{5}+1}}{\frac{3}{5}+1} = \boxed{\frac{5}{8}x^{8/5} + C}$

Rewrite

Formula $\int x^n\,dx = \frac{x^{n+1}}{n+1} + C$ $(n \neq -1)$

b $\int \frac{(4+\sqrt{x})^2}{x}\,dx = \int \frac{16 + 8\sqrt{x} + x}{x}\,dx$

Rewrite $= \int \frac{16}{x}\,dx + \int 8x^{-1/2}\,dx + \int 1\,dx$

Antideri. Formula $= 16\ln|x| + 8\cdot\frac{x^{1/2}}{1/2} + x + C$

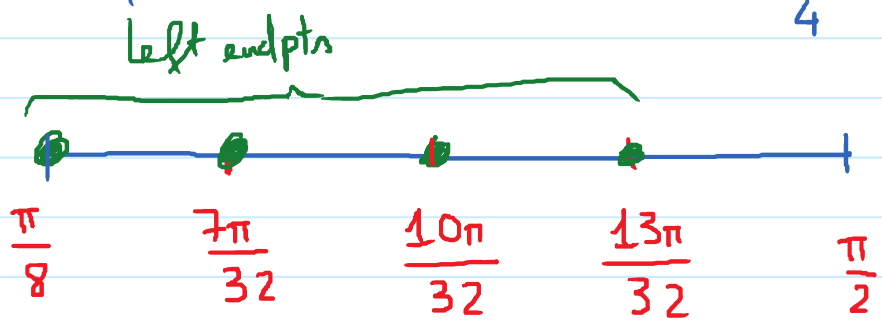
$$= \boxed{16\ln|x| + 16\sqrt{x} + x + C}$$

#6 $f(x) = \cos^2(x)$ on $\left[\frac{\pi}{8}, \frac{\pi}{2}\right]$

4 subintervals.

Width of each subinterval $= \Delta x = \dfrac{\frac{\pi}{2} - \frac{\pi}{8}}{4} = \dfrac{3\pi}{32}$

L_4:



$$L_4 = \underbrace{\frac{3\pi}{32}}_{\text{width}} \underbrace{\left(\cos^2\left(\frac{\pi}{8}\right) + \cos^2\left(\frac{7\pi}{32}\right) + \cos^2\left(\frac{10\pi}{32}\right) + \cos^2\left(\frac{13\pi}{32}\right)\right)}_{\text{heights}}$$

$= \ldots$ (calculator)

R_4

$$R_4 = \underbrace{\frac{3\pi}{32}}_{\text{width}} \underbrace{\left(\cos^2\left(\frac{7\pi}{32}\right) + \cos^2\left(\frac{10\pi}{32}\right) + \cos^2\left(\frac{13\pi}{32}\right) + \cos^2\left(\frac{\pi}{2}\right)\right)}_{\text{heights}}$$

$= \ldots$ (calculator)