#7 Given:

$$\int_0^4 f = 9; \quad \int_0^2 f = -7; \quad \int_0^4 g = -1; \quad \int_0^2 g = 4.$$

Find $\displaystyle\int_0^2 (2\cdot f - 3\cdot g) = 2\int_0^2 f - 3\int_0^2 g = 2\cdot(-7) - 3\cdot 4$

$= \boxed{-26}$

What if we want: $\displaystyle\int_2^4 (2f - 3g) = 2\int_2^4 f - 3\int_2^4 g$

$$\int_2^4 f = \int_0^4 f - \int_0^2 f = 9 - (-7) = 16$$

$$\int_2^4 g = \int_0^4 g - \int_0^2 g = -1 - 4 = -5$$

$2\cdot 16 - 3(-5) = \boxed{47}$.

#8 (a) $F(x) = \int_1^{e^x} \ln(u^2)\, du$

FTC-Part I

$F'(x) \overset{\text{FTC-Part I}}{=} \ln\left((e^x)^2\right) \cdot (e^x)'$

$= \ln\left(e^{2x}\right) \cdot e^x$

$= \boxed{2x \cdot e^x}$

$\frac{d}{dx}\left(\int_a^{f(x)} g(t)\,dt \right)$

$= g(f(x)) \cdot f'(x)$

(FTC - I)

(b) $\int_0^1 \frac{x^2 - 5}{1 + x^2}\, dx = \int_0^1 \frac{x^2 + 1 - 6}{1 + x^2}\, dx$

$= \int_0^1 \frac{x^2+1}{x^2+1}\, dx - 6 \int_0^1 \frac{1}{x^2+1}\, dx$

1

FTC-II

$= x \Big|_0^1 - 6 \arctan(x) \Big|_0^1$

$= (1 - 0) - 6\left(\arctan(1) - \arctan(0)\right)$

$= 1 - 6 \cdot \frac{\pi}{4} = \boxed{1 - \frac{3\pi}{2}}$

#9 (a) $\int \boxed{t^4} \cos^2(t^5) \boxed{\sin(t^5)}\, dt$

Let $u = \cos(t^5)$; $du = -\sin(t^5) \cdot 5t^4\, dt$

$$-\frac{1}{5} \int \Big(\underbrace{\boxed{\cos(t^5)}}_{u}\Big)^2 \underbrace{\left(-\sin(t^5) \cdot 5t^4\right) dt}_{du}$$

$$= -\frac{1}{5} \int u^2\, du = -\frac{1}{5} \cdot \frac{u^3}{3} + C = -\frac{u^3}{15} + C$$

$$= \boxed{-\frac{\cos^3(t^5)}{15} + C}$$

(b) $\int \underbrace{\boxed{\cos(\theta)}}_{1-u} \Big(\underbrace{\boxed{1 - \cos(\theta)}}_{u}\Big)^{99} \underbrace{\boxed{\sin(\theta)\, d\theta}}_{du}$

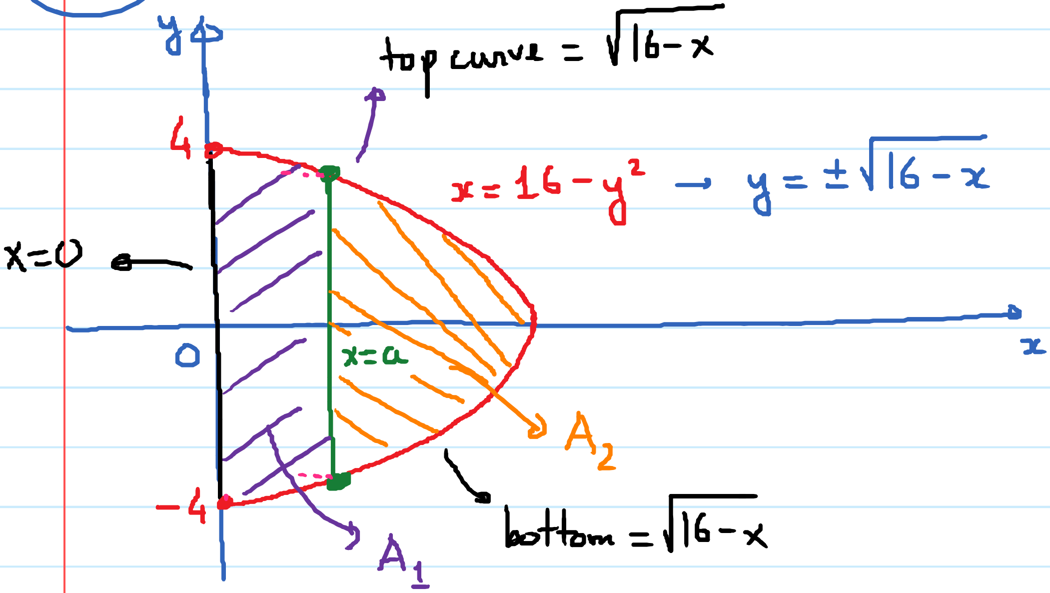
Let $u = 1 - \cos(\theta)$. Then $du = \sin(\theta)\, d\theta$

$\cos\theta = 1 - u$

$$\int (1-u)\, u^{99}\, du = \int \left(u^{99} - u^{100}\right) du$$

$$= \frac{u^{100}}{100} - \frac{u^{101}}{101} + C = \frac{(1-\cos\theta)^{100}}{100} - \frac{(1-\cos\theta)^{101}}{101} + C.$$

#10 $y^2 = 16 - x \rightarrow x = 16 - y^2$



Find a so that $A_1 = A_2$ or $A_1 = \frac{1}{2}$ total area.

$$A_1 = \int_0^a \Big(\underbrace{\sqrt{16-x}}_{\text{top}} - \underbrace{\left(-\sqrt{16-x}\right)}_{\text{bottom}} \Big)\, dx$$

$$= 2\int_0^a \sqrt{16-x}\, dx$$

$$= \frac{-4(16-x)^{3/2}}{3} \Bigg|_0^a$$

$$= -\frac{4}{3}\left[(16-a)^{3/2} - (16)^{3/2}\right]$$

$$= -\frac{4}{3}\left[(16-a)^{3/2} - 64\right]$$

let $u = 16 - x$.

$du = -dx$

$2\int \sqrt{u}\,(-du)$

$= -2\int u^{1/2}\,du = -2\,\frac{u^{3/2}}{3/2}$

$= -\frac{4u^{3/2}}{3}$.

$$\text{Total area} = \int_{-4}^{4} (16 - y^2)\,dy$$

$$= \left(16y - \frac{y^3}{3}\right)\Bigg|_{-4}^{4}$$

$$= \left(64 - \frac{64}{3}\right) - \left(-64 + \frac{64}{3}\right)$$

$$= 128 - \frac{128}{3} = \frac{256}{3}.$$

Want $A_1 = \frac{1}{2}$ total area.

$$\underbrace{-\frac{4}{3}\left[(16-a)^{3/2} - 64\right]}_{A_1} = \frac{1}{2}\cdot\overbrace{\left(\frac{256}{3}\right)}^{\text{total area}}$$

$$-\frac{4}{3}(16-a)^{3/2} + \frac{256}{3} = \frac{128}{3}$$

$$-\frac{4}{3}(16-a)^{3/2} = -\frac{128}{3}$$

$$(16-a)^{3/2} = -\frac{128}{\cancel{3}}\cdot\left(-\frac{\cancel{3}}{4}\right)$$

$$(16-a)^{3/2} = 32 \longrightarrow 16 - a = (32)^{2/3}$$