Name: ________________________

Student ID: ________________________

Section: ________________________

Instructor: ________________________

Math 2413 (Calculus I) Practice Exam 1

Instructions:

- Each question is worth 5 points.
- Work on scratch paper will not be graded.
- No partial credit will be given for the multiple choice part and the short answer part.
- For questions 13 to 16, show all your work in the space provided. Full credit will be given only if the necessary work is shown justifying your answer.
- **Please write neatly. If I cannot read your handwriting, you will not receive credit.**
- Simplify your answers as much as possible. Expressions such as $\ln(1)$, e^0, $\sin(\pi/2)$, etc. must be simplified for full credit.

Multiple Choice. Choose the correct answer for each question. Select one choice only.

1. For the function f whose graph is given in figure 1 below. Find the quantity

$$\lim_{x\to -3} f(x) + \lim_{x\to 0^+} f(x).$$

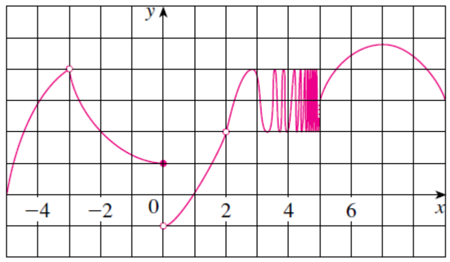

Figure 1: Figure for Question 2

a) -3 b) 3 c) 4

d) 5 e) 0

2. Find the points at which the function is discontinuous and classify the type of discontinuity at each point for the function

$$g(t) = \frac{t+5}{t^2+9t+20}$$

a) Removable discontinuity at $t = -4$ and $t = -5$

b) Removable discontinuity at $t = -4$, Infinite discontinuity at $t = -5$

c) Removable discontinuity at $t = -5$, Infinite discontinuity at $t = -4$

d) Removable discontinuity at $t = -4$, Jump discontinuity at $t = -5$

e) Removable discontinuity at $t = -5$, Jump discontinuity at $t = -4$

3. Find the value of the constant c such that the function f is continuous on the interval $(-\infty, \infty)$.

$$f(x) = \begin{cases} cx^2 + 2x & \text{if } x < 2 \\ x^3 - cx & \text{if } x \geq 2. \end{cases}$$

a) $c = 2$ b) $c = 1$ c) $c = \frac{2}{3}$

d) $c = \frac{1}{3}$ e) $c = -2$

4. The graph of a function f is given in figure 2 below. State all the numbers at which f has a removable discontinuity. Choose the best answer.

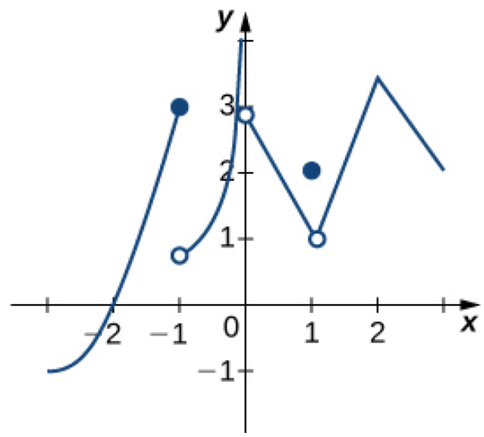

Figure 2: Figure for Question 4

a) -1 b) 0 c) 1

d) $-1, 0$ e) $-1, 1$

5. The limit represents the derivative of a function f at a number a. Find f and a.

$$\lim_{h \to 0} \frac{\cos(\pi + h) + 1}{h}.$$

a) $f(x) = \cos(x + \pi)$, $a = 0$ b) $f(x) = \cos(x + \pi)$, $a = \pi$

c) $f(x) = \sin(x + \pi)$, $a = 0$ d) $f(x) = \cos(x)$, $a = 0$

e) $f(x) = \cos(x)$, $a = \pi$

6. The tangent line to the curve $y = f(x)$ at $(4, 3)$ passes through the point $(0, 2)$. Find $f'(4)$.

a) $\frac{1}{4}$ b) 2 c) 4

d) -4 e) $-\frac{1}{4}$

7. The graph of a function g is given in figure 3 below. Arrange the following numbers in increasing order.

$$0 \qquad g'(-2) \qquad g'(0) \qquad g'(2) \qquad g'(4)$$

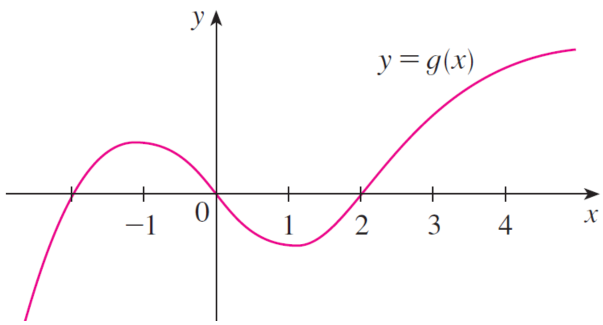

Figure 3: Figure for Question 12

a) $0 < g'(-2) < g'(0) < g'(2) < g'(4)$

b) $g'(4) < g'(2) < g'(0) < g'(-2) < 0$

c) $g'(-2) < 0 < g'(0) < g'(2) < g'(4)$

d) $g'(0) < 0 < g'(-2) < g'(2) < g'(4)$

e) $g'(0) < 0 < g'(4) < g'(2) < g'(-2)$

8. The graphs of the functions F and G are given in figure 4 below. If $H(x) = \dfrac{F(x)}{G(x)}$, find $H'(7)$.

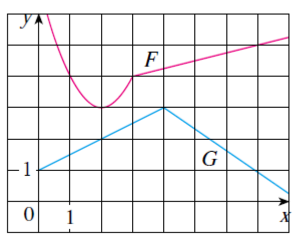

Figure 4: Figure for Question 8

a) $\dfrac{39}{12}$

b) $-\dfrac{41}{12}$

c) $\dfrac{41}{12}$

d) $-\dfrac{43}{12}$

e) $\dfrac{43}{12}$

Short Answer: Write your final answer clearly for each question. No work will be graded. No partial credit.

9. (5 points) Find an equation of the tangent line to the curve $y = \sqrt[4]{x}$ at the point $(1, 1)$.

   Answer Only:

10. (5 points) Find an equation of the tangent line to the graph of the function

$$f(x) = 4\sin(x) + 6\cos(x)$$

   at $x = 0$.

   Answer Only:

11. (5 points) Find $f''(x)$ for the function $f(x) = x + \dfrac{1}{x}$.

   Answer Only:

12. (5 points) Find $\dfrac{dy}{dx}$ for the function $y = \dfrac{x^2 + 4x + 3}{\sqrt{x}}$.

   Answer Only:

Essay: Show all work in the space provided. Full credit will be given only if all steps are shown justifying your answer. Please write neatly and carefully, if I cannot read your handwriting, you will receive NO credit. Scratch work will not be graded.

13. (10 points) Find the given limit **analytically**. (No credit will be given if you find the limit by making a table of values or graphing).

$$\lim_{x \to -4} \frac{\sqrt{x^2+9}-5}{x+4}$$

14. (10 points) Let $f(x) = 4x - 3x^2$.

    (a) Find $f'(2)$ using the **definition of the derivative**. (**Any other method will receive NO credit**)

(b) Use the result from the previous part to find an equation of the tangent line to the graph of $y = f(x)$ at the point where $x = 2$.

15. (10 points) Find the x values for which the tangent line to the graph of $f(x) = x - 2\cos(x)$, $0 < x < 2\pi$ has slope 2.

16. (10 points) Find the derivative with respect to x of the function

$$f(x) = \frac{c^8 - x^8}{c^8 + x^8}, c \text{ is a constant.}$$

END OF EXAM