Name:_____

Student ID:_____

Section:_____

Instructor:_____

Math 2413 (Calculus I)
Practice Exam 2

Instructions:

- Each multiple choice question is worth 5 points.

- Work on scratch paper will not be graded.

- No partial credit will be given for the multiple choice part and the short answer part.

- For questions 13 to 16, show all your work in the space provided. Full credit will be given only if the necessary work is shown justifying your answer.

- **Please write neatly. If I cannot read your handwriting, you will not receive credit.**

- Simplify your answers as much as possible. Expressions such as $\ln(1)$, e^0, $\sin(\pi/2)$, etc. must be simplified for full credit.

Multiple Choice. Choose the correct answer for each question. Select one choice only.

1. A particle moves according to the position function $s(t) = 3 - \dfrac{5}{t^2}$, $t > 0$. Find its acceleration function.

   a) $\dfrac{10}{t^3}$　　　　　　　b) $-\dfrac{10}{t^3}$　　　　　　　c) $\dfrac{15}{t^4}$

   d) $-\dfrac{15}{t^4}$　　　　　　　e) $\dfrac{30}{t^4}$　　　　　　　f) $-\dfrac{30}{t^4}$

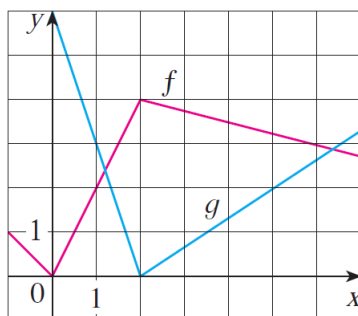2. Let $u(x) = f(g(x))$ where the graph of f and g are shown in figure 1. Find $u'(1)$.

Figure 1: Figure for Question 2

   a) -6　　　　　　　b) 6　　　　　　　c) $-\dfrac{2}{3}$

   d) $\dfrac{2}{3}$　　　　　　　e) $-\dfrac{3}{4}$　　　　　　　f) $\dfrac{3}{4}$

3. Let $y = (f(u) + 4x)^2$ and $u = x^3 - 2x$. If $f(4) = 5$ and $\dfrac{dy}{dx} = 13$ when $x = 2$, find $f'(4)$.

   a) $-\dfrac{7}{2}$　　　　　　　b) $\dfrac{7}{2}$　　　　　　　c) $-\dfrac{7}{20}$

   d) $\dfrac{7}{20}$　　　　　　　e) $-\dfrac{7}{10}$　　　　　　　f) $\dfrac{7}{10}$

4. Let $f(x) = x^5 + 3x^3 - 4x - 5$. Find the slope of the tangent line to the graph of its inverse function f^{-1} at the point $P(-5, 1)$ on the graph of f^{-1}.

   a) $-\dfrac{1}{10}$　　　　　　　b) $\dfrac{1}{10}$　　　　　　　c) $-\dfrac{1}{3346}$

   d) $\dfrac{1}{3346}$　　　　　　　e) $-\dfrac{1}{9}$　　　　　　　f) $\dfrac{1}{9}$

5. Find the equation of the tangent line to the graph of the equation $x^2y^2 + 5xy = 36$ at the point $(4, 1)$.

    a) $y = 4x - 15$ b) $y = -4x + 17$ c) $y = \dfrac{1}{4}x$

    d) $y = -\dfrac{1}{4}x + 2$ e) $y = 5x - 19$ f) $y = \dfrac{1}{5}x + \dfrac{1}{5}$

6. Let $f(x) = 4^{\sin(4x)}$. Find $f'(x)$

    a) $4^{\sin(4x)}\ln(4)$ b) $4^{\cos(4x)}\ln(4)$ c) $4^{\sin(4x)}\ln(4)\cos(4x)$

    d) $4^{\cos(4x)}\ln(4)$ e) $4^{\sin(4x)+1}\ln(4)\cos(4x)$ f) $4^{\sin(4x)}\ln(16)\cos(4x)$

7. The base of a triangle is shrinking at a rate of 1 cm/min and the height of the triangle is increasing at a rate of 3 cm/min. Find the rate (in cm^2/min) at which the area of the triangle changes when the height is 34 cm and the base is 18 cm.

    a) $5\text{cm}^2/\text{min}$ b) $10\text{cm}^2/\text{min}$ c) $15\text{cm}^2/\text{min}$

    d) $20\text{cm}^2/\text{min}$ e) $25\text{cm}^2/\text{min}$ f) $30\text{cm}^2/\text{min}$

8. (Section 4.2) Given $y = \sqrt{x}$, $x = 1$, $\Delta x = 1$. Find Δy and dy

    a) $\Delta y = dy = 0.414$ b) $\Delta y = dy = 0.5$ c) $\Delta y = 0.414$, $dy = 0.5$

    d) $\Delta y = 0.5$, $dy = 0.414$ e) $\Delta y = 1.414$, $dy = 1$ f) $\Delta y = 1$, $dy = 1.414$

Short Answer: Write your final answer clearly for each question. No work will be graded. No partial credit.

9. (5 points) Find the derivative of the function $y = \sin(\sin(\sin(x)))$.

   Answer Only:

10. (5 points) Find the derivative of the function $y = \sin^{-1}\left(\sqrt{\sin x}\right)$.

    Answer Only:

11. (5 points) Find the x-value of the point in the first quadrant at which the tangent line to the curve $x^3 + y^3 = 6xy$ is horizontal.

    Answer Only:

12. (5 points) Let $f(x) = \log_a(3x^2 - 2)$. Find a such that $f'(1) = 3$.

    Answer Only:

Essay: Show all work in the space provided. Full credit will be given only if all steps are shown justifying your answer. Please write neatly and carefully, if I cannot read your handwriting, you will receive NO credit. Scratch work will not be graded.

13. (10 points) A 16 ft ladder is leaning against a wall. If the top of the ladder slides down the wall at a rate of 2 ft/s, how fast (in ft/s) is the bottom moving along the ground when the bottom of the ladder is 8 ft from the wall?
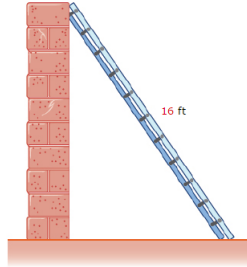

16 ft

Figure 2: Figure for Question 13

14. (10 points) Find the linear approximation of $f(x) = x^6$ at $x = 2$ and use the approximation to estimate $(2.001)^6$.

15. (10 points) Find $\dfrac{dy}{dx}$ for $y = (\ln x)^{\cos x}$.

16. (10 points) The given function represents the position of a particle traveling along a horizontal line.
$$s(t) = \frac{t}{4 + t^2}, t \geq 0.$$

Find the velocity and acceleration functions (of course, you can use any method you like to find the derivative) and determine the time intervals when the object is slowing down or speeding up.