Review 2

Monday, March 4, 2019 8:57 AM

#2 $\left[\underbrace{f(g(x))}_{u(x)}\right]' = f'(g(x))\cdot g'(x)$

Plug $x=1 \longrightarrow u'(1) = \overbrace{\boxed{f'(\underbrace{\boxed{g(1)}}_{3})}}^{\left(-\frac{1}{4}\right)} \cdot \overbrace{\boxed{g'(1)}}^{(-3)} = \dfrac{3}{4}$

$u'(1) = \dfrac{3}{4}$

function of x

outside

#3 $y = (\underbrace{f(u) + 4x}_{\text{inside}})^{\boxed{2}}$

$$\frac{dy}{dx} = 2\cdot(f(u)+4x)\cdot(f'(u)\cdot u' + 4)$$

$$\frac{dy}{dx} = 2\cdot(f(x^3-2x)+4x)\cdot(f'(x^3-2x)\cdot(3x^2-2)+4)$$

Plug $x=2$ to both sides:

$$\boxed{13} = 2\cdot(\underbrace{\boxed{f(4)}}_{5}+8)\cdot(\overbrace{\boxed{f'(4)}}^{?}\cdot 10 + 4)$$

$\left.\dfrac{dy}{dx}\right|_{x=2}$

$$13 = 2\cdot(5+8)\cdot(10\overbrace{\boxed{f'(4)}}^{?}+4)$$

$$\frac{1}{2} = 10f'(4)+4 \rightarrow -\frac{7}{2} = 10f'(4)$$

$$\boxed{f'(4) = -\frac{7}{20}}$$

4

$$\underbrace{(f^{-1})'(-5)}_{\text{Slope}} = \frac{1}{f'(f^{-1}(-5))} = \frac{1}{f'(1)}$$

$\boxed{f^{-1}(-5) = 1}$ (Pt $P(-5,1)$ is on graph of f^{-1})

$f'(x) = 5x^4 + 9x^2 - 4 \rightarrow f'(1) = 10$

$$(f^{-1})'(-5) = \boxed{\frac{1}{10}}$$

5 $x^2y^2 + 5xy = 36$

$$2xy^2 + x^2 \cdot 2y \cdot \frac{dy}{dx} + 5y + 5x\frac{dy}{dx} = 0$$

Plug point $(4, 1)$ into equation and find $\frac{dy}{dx}$

x y

$$8 + 32\frac{dy}{dx} + 5 + 20\frac{dy}{dx} = 0$$

$$\rightarrow \frac{dy}{dx} = -\frac{1}{4} \rightarrow \text{Slope.}$$

Monday, March 4, 2019 9:20 AM

Pt-slope: $y - 1 = -\frac{1}{4}(x - 4)$

$$\boxed{y = -\frac{1}{4}x + 2}$$

#6

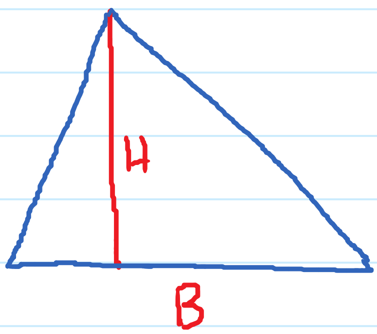
Formula: $(a^u)' = a^u \cdot \ln a \cdot u'$

$a = 4$; $u = \sin(4x)$

$$\left((4)^{\sin(4x)}\right)' = 4^{\sin(4x)} \cdot \ln(4) \cdot \cos(4x) \cdot 4^1$$

$$= 4^{\sin(4x)+1} \cdot \ln(4) \cdot \cos(4x)$$

#7



Quantities changing with time

B (base)

H (height)

A (area)

$A = \frac{1}{2}BH$ → take derivative w.r.t. t

$$\frac{dA}{dt} = \frac{1}{2}\left(\underset{34}{\boxed{H}}\underset{-1}{\boxed{\frac{dB}{dt}}} + \underset{18}{\boxed{B}}\underset{3}{\boxed{\frac{dH}{dt}}}\right) = \boxed{10 \ cm^2/min}$$

Monday, March 4, 2019 9:33 AM

$\boxed{\#8}$ $f(x)=\sqrt{x}$; $x=1$, $\Delta x=1$

Find Δy and dy.

$$\Delta y = f(x+\Delta x) - f(x)$$

$$\Delta y = f(2) - f(1) = \sqrt{2} - 1 \simeq 0.414.$$

$$dy = f'(x)\,dx \quad . \quad (dx = \Delta x = 1)$$

$$dy = f'(1)\cdot 1 = f'(1). \qquad f'(x) = \frac{1}{2\sqrt{x}}$$

So, $dy = \dfrac{1}{2} = 0.5$

$\boxed{\#9}$

$$y' = \cos(\sin(\sin x))\cdot\cos(\sin x)\cdot\cos x$$

$\boxed{\#10}$

$$y = \sin^{-1}\big(\underbrace{\boxed{\sqrt{\sin x}}}_{u}\big)$$

Formula: $(\arcsin u)' = \dfrac{u'}{\sqrt{1-u^2}}$

$$y' = \frac{(\sqrt{\sin x})'}{\sqrt{1-\sin x}} = \frac{\frac{\cos x}{2\sqrt{\sin x}}}{\sqrt{1-\sin x}} = \frac{\cos x}{2\sqrt{\sin x}\cdot\sqrt{1-\sin x}}$$

$$= \boxed{\frac{\cos x}{2\sqrt{\sin x(1-\sin x)}}}$$

#11 $x^3 + y^3 = 6xy$

$$3x^2 + 3y^2 \frac{dy}{dx} = 6y + 6x\frac{dy}{dx}$$

$$x^2 + y^2 \frac{dy}{dx} = 2y + 2x\frac{dy}{dx}$$

$$(y^2 - 2x)\frac{dy}{dx} = 2y - x^2$$

$$\frac{dy}{dx} = \frac{2y - x^2}{y^2 - 2x}$$

Horizontal tangent line $\rightarrow \frac{dy}{dx} = 0 \rightarrow 2y - x^2 = 0$

$\rightarrow x^2 = 2y.$

$$\begin{cases} x^3 + y^3 = 6xy \\ x^2 = 2y \rightarrow y = \frac{x^2}{2} \end{cases} \longrightarrow x^3 + \left(\frac{x^2}{2}\right)^3 = 6x \cdot \frac{x^2}{2}$$

$\rightarrow x^3 + \frac{x^6}{8} = 3x^3$

$\rightarrow -2x^3 + \frac{x^6}{8} = 0$

$\rightarrow x^3\left(-2 + \frac{x^3}{8}\right) = 0$ (First quadrant)

$x^3 = 0$; $-2 + \frac{x^3}{8} = 0$

$\cancel{x = 0}$; $\frac{x^3}{8} = 2 \rightarrow x^3 = 16 \rightarrow \boxed{x = \sqrt[3]{16}}$

$\boxed{\#12}$ $f(x) = \log_a(\underbrace{\boxed{3x^2-2}}_{u})$

Find a s.t. $f'(1) = 3$

<u>Formula:</u> $\left(\log_a(u)\right)' = \dfrac{u'}{u\ln a}$

$$f'(x) = \frac{6x}{(3x^2-2)\ln a}$$

$$f'(1) = \frac{6}{\ln a} = 3$$

$$\longrightarrow \ln a = 2 \rightarrow \boxed{a = e^2}$$

$\boxed{\#13}$ HW problem (Did in class)

$\boxed{\#14}$ $\boxed{L(x) = 192(x-2) + 64}$.

$$L(2.001) = 192\cdot(0.001) + 64 = \boxed{64.192}$$

$\boxed{\#15}$ $y = (\ln x)^{\cos x}$

$$\longrightarrow \ln y = \ln\left((\ln x)^{\boxed{\cos x}}\right)$$

$$\longrightarrow \ln y = \cos x \cdot \ln(\ln x)$$

$$\frac{1}{y}\frac{dy}{dx} = (-\sin x)\cdot \ln(\ln x) + \cos x \cdot \frac{1}{\ln x}\cdot\frac{1}{x}$$

$$\frac{dy}{dx} = \left[-\sin x \cdot \ln(\ln x) + \frac{\cos x}{x \ln x}\right]\cdot \underbrace{(\ln x)^{\cos x}}_{y}$$

$\boxed{\#16}$ Process: $v(t) = s'(t)$; $a(t) = s''(t) = v'(t)$

Set $v=0$; $a=0$. Make table, determine signs.

Speed up: v and a have same sign.

Slow down: v and a have different signs.