Test 3 Review

Monday, April 8, 2019 8:05 AM

(1) Domain and critical numbers

$$f(x) = 2x^3 + \frac{3}{x}$$

Domain: Set Denom $= 0$

$$x = 0.$$

Domain: $(-\infty, 0) \cup (0, \infty)$

Critical numbers: Find $f'(x)$ — Set $= 0$ / when undefined

$$f'(x) = 6x^2 - \frac{3}{x^2}$$

* Undefined when $x = 0$. But $x = 0$ is not in the domain.

So, $x = 0$ is not a critical #.

* $f'(x) = 0 \longrightarrow \frac{x^2 \cdot 6x^2}{x^2 \cdot 1} - \frac{3}{x^2} = 0$

$$\longrightarrow \frac{6x^4 - 3}{x^2} = 0$$

$$\longrightarrow 6x^4 - 3 = 0 \longrightarrow x^4 = \frac{1}{2} \longrightarrow x = \pm\sqrt[4]{\frac{1}{2}}$$

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Conclusion: Critical #s are : $\sqrt[4]{\frac{1}{2}}$; $-\sqrt[4]{\frac{1}{2}}$

(2) $\boxed{\text{Abs. Max / Min}}$ of $f(x) = \dfrac{4x^2}{x-2}$ on $[-2, 1]$

Step 1: Find critical numbers in $[a, b]$

Step 2: Find values of function at critical #s and at endpoints and compare.

* Step 1:

$$f'(x) = \frac{8x \cdot (x-2) - 4x^2 \cdot 1}{(x-2)^2}$$

$$f'(x) = \frac{8x^2 - 16x - 4x^2}{(x-2)^2}$$

$$f'(x) = \frac{4x^2 - 16x}{(x-2)^2}$$

$f'(x) = 0 \longrightarrow 4x^2 - 16x = 0 \longrightarrow 4x(x-4) = 0$

$\longrightarrow x = 0$; $x = 4$ $\longrightarrow$ is not in $[-2, 1]$

* Step 2:

important values of x

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	x	$f(x)$
endpt	-2	-4
critical #	0	0
endpt	1	-4

Conclusion:

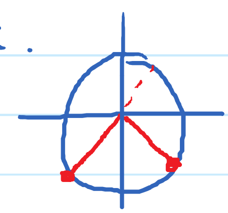
$\boxed{\text{Abs. max} = 0}$ when $x = 0$

$\boxed{\text{Abs. min} = -4}$ when $x = -2$ or $x = 1$

③ Increasing | Decreasing Intervals

→ 1^{st} derivative test.

$f(x) = x - 2\cos(x)$; $0 < x < 2\pi$.



$f'(x) = 1 + 2\sin(x) = 0$

$\rightarrow \sin(x) = -\frac{1}{2} \rightarrow x = \frac{7\pi}{6}, \frac{11\pi}{6}$.

→ Make a sign chart:

$\frac{3\pi}{2} = \frac{9\pi}{6}$; $\frac{11.5\pi}{6}$

x: 0 Test pt: $\frac{\pi}{6}$ $\frac{7\pi}{6}$ Test pt: $\frac{9\pi}{6}$ $\frac{11\pi}{6}$ Test pt 2π

f': $+$ $-$ $+$

f is increasing on: $\left(0, \frac{7\pi}{6}\right) \cup \left(\frac{11\pi}{6}, 2\pi\right)$

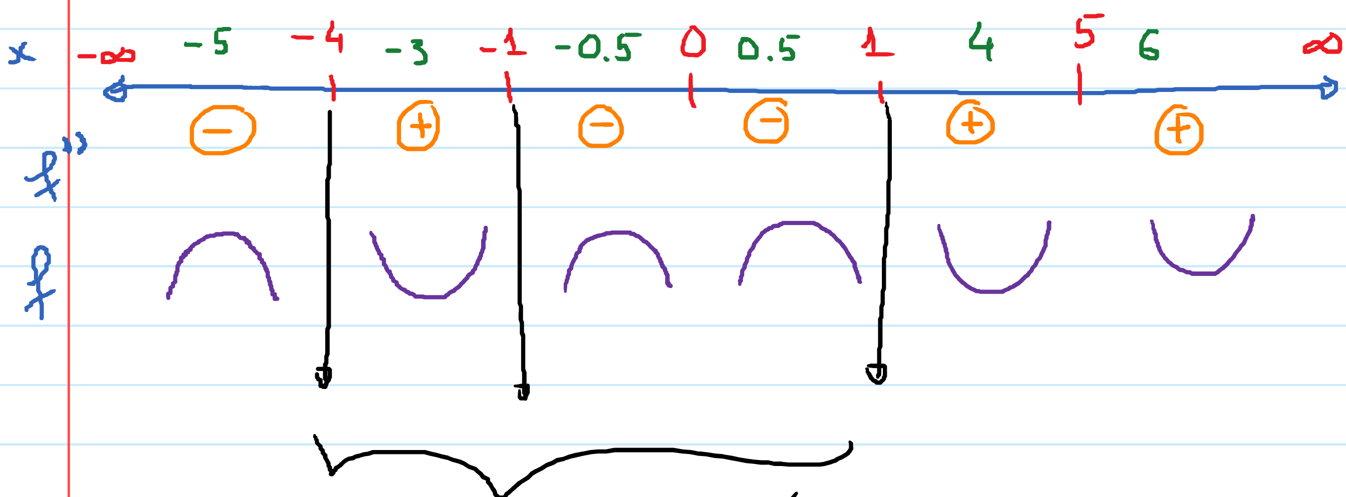
f is decreasing on: $\left(\frac{7\pi}{6}, \frac{11\pi}{6}\right)$

(4) Given: $f''(x) = x^2 (x-5)^4 (x+4)^3 (x^2-1)$

(a) Inflection points:

$f''(x) = 0 \rightarrow x = 0 \; ; \; x-5=0 \; ; \; x+4=0 \; ; \; x^2-1=0$

$x = 5 \; ; \; x = -4 \; ; \; x = \pm 1$



Inflection points (Concavity changes)

Conclusion: x-values of inflection points: $-4, -1, 1$.

(b) Given: $f' = 0$ at $-5, \frac{1}{2}, 0, 4, -2$.

which corresponds to a max / min.

Plug these points into f''.

$\left.\begin{array}{l} f'(c) = 0 \\ f''(c) > 0 \end{array}\right\} \rightarrow$ local min $\qquad \left.\begin{array}{l} f'(c) = 0 \\ f''(c) < 0 \end{array}\right\} \rightarrow$ local max

$f'(c) = f''(c) = 0 \to$ test fails.

x	f'	f''	
-5	0	$-$	$\to$ local max at $x = -5$
$\frac{1}{2}$	0	$-$	$\to$ local max at $x = \frac{1}{2}$
0	0	0	$\to$ inconclusive
4	0	$+$	$\to$ local min at $x = 4$
-2	0	$+$	$\to$ local min at $x = -2$.

(given

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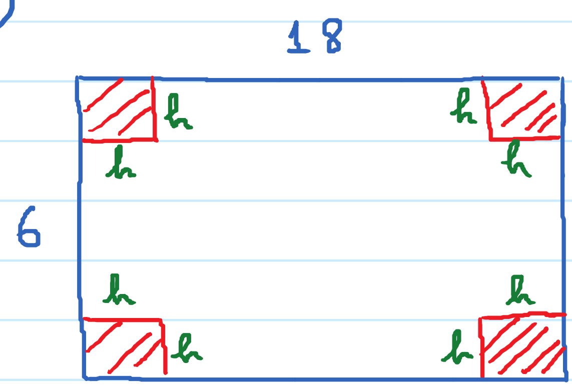

length of box : $18 - 2h$

width of box : $6 - 2h$

Height of box : h.

Volume of box $= V(h) = (18-2h)(6-2h)\,h$.

Constraint for h: $0 \le h \le 3$

$\longrightarrow$ Find abs. max of $V(h)$ on $[0,3]$

$\longrightarrow$ Optimization problem on closed interval.