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$$V(h) = (18-2h)(6-2h)h$$
$$= (18-2h)(6h-2h^2)$$
$$= 108h - 36h^2 - 12h^2 + 4h^3$$
$$V(h) = 4h^3 - 48h^2 + 108h$$
$$V'(h) = 12h^2 - 96h + 108 = 0$$
$$12(h^2 - 8h + 9) = 0$$

Quadratic formula: $h = \dfrac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

$$h = \frac{8 \pm \sqrt{28}}{2} = \frac{8 \pm 2\sqrt{7}}{2}$$

$h = 4 \pm \sqrt{7}$ $\rightarrow$ $4+\sqrt{7}$ is outside of $[0,3]$

h	$V(h)$
0	0
$4-\sqrt{7}$	68.16 $\rightarrow$ abs. max
3	0

So, $\boxed{h = 4-\sqrt{7}}$

$$\boxed{l = 18 - 2(4-\sqrt{7})\ ;\ w = 6 - 2(4-\sqrt{7})}$$

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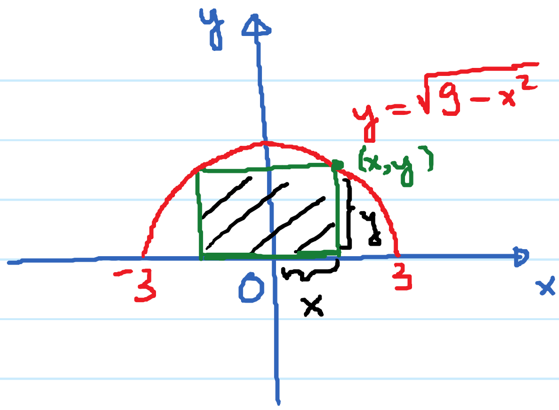
6



Area of rectangle $= 2xy = 2x \cdot \sqrt{9-x^2}$

Constraint for x: $0 \leq x \leq 3$

$\to$ Max. $A(x) = 2x \cdot \sqrt{9-x^2}$ on $[0, 3]$.

$$A'(x) = 2 \cdot \sqrt{9-x^2} + \cancel{2}x \cdot \frac{-2x}{\cancel{2}\sqrt{9-x^2}}$$

$$= \frac{2 \cdot \sqrt{9-x^2}}{1} - \frac{2x^2}{\sqrt{9-x^2}}$$

$$= \frac{2(9-x^2) - 2x^2}{\sqrt{9-x^2}} = \frac{18-4x^2}{\sqrt{9-x^2}}$$

outside

$A'(x)$ is undefined when $x=-3$; $\boxed{x=3} \to$ critical #

$A'(x) = 0$ when $18-4x^2 = 0 \to x^2 = \frac{18}{4} \to$

outside

$x = -\frac{3\sqrt{2}}{2}$ or $x = \frac{3\sqrt{2}}{2}$ critical #

x	$A(x)$
0	0
$\frac{3\sqrt{2}}{2}$	9
3	0

max area.

→ abs. max

Area is max when $x = \frac{3\sqrt{2}}{2}$ and $y = \frac{3\sqrt{2}}{2}$.

7 $\lim\limits_{x \to 0} \dfrac{4x + e^{x} - e^{5x}}{1 - \cos(4x)}$ $\left(\frac{0}{0}\right)$ → L'Hopital

$= \lim\limits_{x \to 0} \dfrac{4 + e^{x} - 5e^{5x}}{4\sin(4x)}$ $\left(\frac{0}{0}\right)$ → L'Hopital

$= \lim\limits_{x \to 0} \dfrac{e^{x} - 25e^{5x}}{16\cos(4x)}$

$= \dfrac{-24}{16} = \boxed{-\frac{3}{2}}$

8) $L = \lim_{x \to 0^+} (1 + x)^{\frac{3}{x}}$ $\quad (1^{\infty})$

$$\ln(L) = \lim_{x \to 0^+} \boxed{\frac{3}{x}} \ln(1 + x)$$

$$= \lim_{x \to 0^+} \frac{3}{x} \cdot \ln(1+x)$$

$$= \lim_{x \to 0^+} \frac{3\ln(1+x)}{x} \quad \left(\frac{0}{0}\right) \to \text{L'Hopital}$$

$$= \lim_{x \to 0^+} \frac{\frac{3}{1+x}}{1} = 3$$

$$\ln(L) = 3 \to \boxed{L = e^3}.$$

9) Find H. A.

a) $f(x) = \dfrac{x^4 - 8x^3 + 1}{7 - 7x^2 - 9x^4}$

$$\lim_{x \to \infty} f(x) = \lim_{x \to -\infty} f(x) = \frac{x^4}{-9x^4} = \frac{1}{-9} = -\frac{1}{9}$$

H. A. $\boxed{y = -\frac{1}{9}}$.

(b) $g(x) = \dfrac{\sqrt{x^2-2}}{2x-7}$

$$\lim_{x\to\infty} \frac{\sqrt{x^2-2}}{2x-7} = \lim_{x\to\infty} \frac{\sqrt{x^2}}{2x} = \frac{x}{2x} = \frac{1}{2}$$

$$\lim_{x\to-\infty} \frac{\sqrt{x^2-2}}{2x-7} = \lim_{x\to-\infty} \frac{\sqrt{x^2}}{2x} = \frac{-x}{2x} = -\frac{1}{2}.$$

So, there are 2 H.A. : $y = \frac{1}{2}$; $y = -\frac{1}{2}$.

(10) $f(x) = x^2 - a.$

(a) $x_{n+1} = x_n - \dfrac{f(x_n)}{f'(x_n)}$

$$\boxed{x_{n+1} = x_n - \frac{x_n^2 - a}{2x_n}}$$

(b) Approximate $\sqrt{10} \to f(x) = x^2 - 10$

$x_0 = 3$; $x_1 = 3 - \dfrac{9-10}{6} = \dfrac{19}{6}$

$x_2 = \dfrac{60}{19} = 3.1579\ldots$

$x_3 = 3.1619\ldots$; $x_4 = 3.162\ldots$