Trig Substitution. Wednesday une 12, 2019 2:10 PM E.g. 1: Substitution: $x = a \sin \Theta$ $\int \frac{4}{x^2 \sqrt{4} - x^2} dx$ let $x = 4 \cdot \sin \Theta \longrightarrow dx = 4 \cos \Theta d\Theta$ $= \int \frac{4}{16 \sin^2 \theta} \sqrt{16 - 16 \sin^2 \theta} 4 \cos \theta \, d\theta$ $= \int \frac{16}{16} \cos \theta \, d\theta$ $= \int \frac{16}{16} \sin^2 \theta \cdot \sqrt{16(1 - \sin^2 \theta)} \, d\theta$ $\cos^2 \theta$ $= \frac{\cos \theta}{\sin^2 \theta \cdot 4 \cdot \cos \theta} d\theta$ $= \frac{1}{4} \left(\frac{1}{\sin^2 \Theta} d\Theta = \frac{1}{4} \left(\csc^2 \Theta d\Theta \right) \right)$ $= -\frac{1}{1} \cot(\theta) + C$

Wednesday, June 12, 2019 2:23 PM

 $x = 4 \sin \theta \rightarrow \sin \theta = \frac{x}{L}$ Solt CAH TOA × Want coto -> coto = adj e $? = \sqrt{|6-x^2|}$ $\cot \Theta = \sqrt{16 - x^2}$ $x^2 + ?^2 = 16$ X $? = \sqrt{16 - x^2}$ $\int \frac{4}{x^2 \sqrt{16 - x^2}} \, dx = -\frac{1}{4} \cdot \frac{\sqrt{16 - x^2}}{x} + C$ $\frac{E_{g.2}}{4\sqrt{4+x^2}} dx$ $x = 2 \tan \theta \longrightarrow dx = 2 \cdot \sec^2 \theta \, d\theta$ $4\int \frac{\$ \tan^3 \theta}{4\sqrt{4+4\tan^2 \theta}} = 2 \sec^2 \theta \, d\theta$ $4\int \frac{\tan^3\theta \sec^2\theta}{\sqrt{4(1+\tan^2\theta)}} d\theta$, Ael20

Wednesday, June 12, 2019 2:31 PM

 $= 2 \left(\frac{\tan^3 \Theta \sec^2 \Theta}{\sec \Theta} \right) d\theta$ $= 2 \left(\frac{1}{\tan^3 \Theta} \operatorname{Aec} \Theta \right) d\Theta$ zdu = 2 $(\tan^2 \theta)$ $\tan \theta \sec \theta d\theta$ let $u = \sec \theta$ du = seco tano do $= 2 \left(\left(u^2 - 1 \right) du \right)$ $= 2 \cdot \left(\frac{u^3}{3} - u\right) + C$ $= 2\left(\frac{NeL^{3}\Theta}{3} - NeL\Theta\right) + C$ 12×4 $\begin{array}{c} 1: \\ x = 2 \tan \theta \rightarrow \tan \theta = \frac{x}{2} \\ \end{array}$ Recall : je χ $\Delta e_{c} \Theta = \frac{\sqrt{x^{2}+4}}{2} + \frac{1}{2} + \frac{1$

N

Vednesday, June 12, 2019 2137 PM

$$\int \frac{x^{3}}{4\sqrt{4 + x^{2}}} dx = 2\left(\frac{(x^{2} + 4)^{2}}{24} - \frac{\sqrt{x^{2} + 4}}{2}\right) + C$$

$$= \frac{8}{4}$$

$$E \cdot g \cdot 3 \cdot \int \sqrt{x^{2} - 16} dx$$

$$= 4 \cdot x = 4 \cdot Aec\theta \rightarrow dx = 4 \cdot Aec\theta + an\theta d\theta$$

$$\int \frac{\sqrt{16 \cdot Aec}^{2}\theta - 16}{\sqrt{4} \cdot Aec\theta} + an\theta d\theta$$

$$\int \frac{\sqrt{16 \cdot Aec}^{2}\theta - 16}{\sqrt{4} \cdot Aec\theta} + an\theta d\theta$$

$$= \frac{4}{4} \cdot \frac{\sqrt{16} \left(\frac{Aec}{2\theta - 1}\right)}{\sqrt{4} \cdot Aec\theta} + an\theta d\theta$$

$$= \int \frac{4}{\sqrt{4} \cdot Aec\theta} + an\theta d\theta = \int \frac{4ea^{2}\theta}{\sqrt{4} \cdot Aec\theta} + aec\theta} d\theta$$

$$= \int \frac{4ea^{2}\theta}{\sqrt{4} \cdot Aec\theta} + aec\theta + aec\theta} d\theta$$

$$= \int \frac{4e^{2}\theta}{\sqrt{4} \cdot Aec\theta} + aec\theta} d\theta$$

$$= \int \frac{4ea^{2}\theta}{\sqrt{4} \cdot Aec\theta} + aec\theta} d\theta$$

$$= \int \frac{4e^{2}\theta}{\sqrt{4} \cdot Aec\theta} + aec\theta} d\theta$$

$$= \int \frac{4ea^{2}\theta}{\sqrt{4} \cdot Aec\theta} + aec\theta} d\theta$$

$$= \int \frac{4e^{2}\theta}{\sqrt{4} \cdot Aec\theta} + aec\theta} d\theta$$

$$= \int \frac{4e^{2}\theta}{\sqrt{4} \cdot Aec\theta} + aec\theta} d\theta$$

$$= \int \frac{4e^{2}\theta}{\sqrt{4} \cdot Aec\theta} + aec\theta} + aec\theta$$

$$= \int \frac{4e^{2}\theta}{\sqrt{4} \cdot Aec\theta} + aec\theta} + aec\theta} + aec\theta$$

$$= \int \frac{4e^{2}\theta}{\sqrt{4} \cdot Aec\theta} + aec\theta} + aec\theta} + aec\theta} + aec\theta$$

$$= \int \frac{4e^{2}\theta}{\sqrt{4} \cdot Aec\theta} + aec\theta} + aec\theta} + aec\theta} + aec\theta$$

$$= \int \frac{4e^{2}\theta}{\sqrt{4} \cdot Aec\theta} + aec\theta} +$$

Wednesday, June 12, 2019 2:45 PM $\pi/3$ $\frac{\tan^2 \Theta}{\operatorname{Nec} \Theta} d\Theta = \int \frac{\operatorname{Nec}^2 \Theta - 1}{\operatorname{Nec} \Theta} d\Theta$ π/3 Archan Archan **d0** nece π 3 (rel - coro) do D = (ln sec + tan 0 - sin0 $\left| 2 + \sqrt{3} \right| - \frac{\sqrt{3}}{2} \right| -$ -ln 2+3 - 13 2 Ξ

Wednesday, June 12, 2019 2:49 PM

 $\frac{E.g.4.}{\sqrt{x^2-6x+5}}dx$ $x^{2}-6x+5 = x^{2}-6x+9 -9+5$ $=(x-3)^2-4$ $\int \frac{x}{\left(x-3\right)^2 - 4} dx$ Let $x-3 = 2 \operatorname{sec} \Theta \longrightarrow dx = 2 \operatorname{sec} \Theta \tan \Theta d\Theta$ $\int \frac{2 \operatorname{Nec} \Theta + 3}{4 \operatorname{Nec}^2 \Theta - 4} 2 \operatorname{Nec} \Theta \tan \Theta \, d\Theta$ $= \int \frac{2 \operatorname{Rec} \Theta + 3}{\sqrt{4 (\operatorname{Rec}^2 \Theta - 1)}} 2 \operatorname{Rec} \Theta + \frac{3}{4 (\operatorname{Rec}^2 \Theta - 1)}$ > tan 20

Wednesday, June 12, 2019 2:55 PM

 $= \frac{2 \operatorname{Rec} \Theta + 3}{2} \cdot \frac{1}{2} \operatorname{Rec} \Theta \, d\Theta$

 $= \left(\left(2 \operatorname{Rec} \Theta + 3 \right) \operatorname{Rec} \Theta \right) \partial \Theta$

= $(2 \sec^2 \Theta + 3 \sec \Theta) d\Theta$

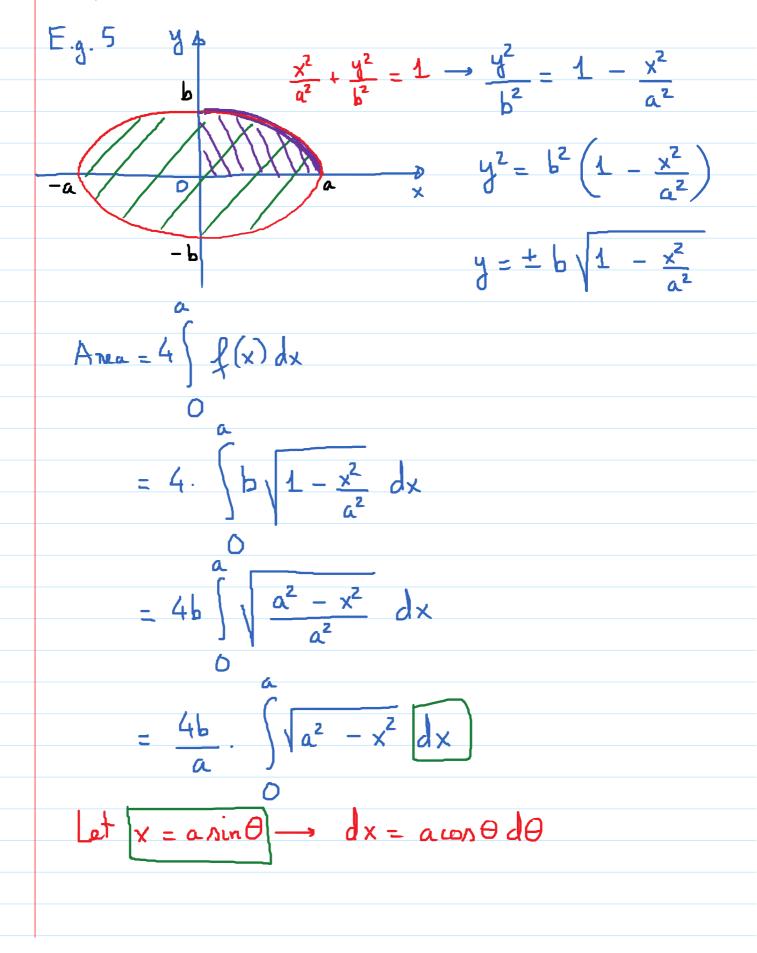
2 tano + 3 ln seco + tano + C

Recall: $x-3 = 2 \operatorname{Rec} \Theta \longrightarrow \operatorname{Rec} \Theta = \frac{x-3}{7}$

 $\frac{x-3}{10}$ $\sqrt{(x-3)^2-4}$ $\frac{1}{100} = \sqrt{(x-3)^2-4}$

 $= \frac{\chi \cdot (x-3)^2 - 4}{2} + 3 \ln \left| \frac{x-3}{2} + \frac{\sqrt{(x-3)^2 + 4}}{2} \right| + C$

Wednesday, June 12, 2019 3:00 PM



Wednesday, June 12, 2019 3:07 PM

$$=\frac{4b}{a}\int \sqrt{a^2 - a^2 \sin^2 \theta} \ a \cos \theta \ d\theta$$

$$=\frac{4b}{c}\int \sqrt{a^2 (1 - \sin^2 \theta)} \ a \cos \theta \ d\theta$$

$$=\frac{4b}{c}\int \sqrt{a^2 (1 - \sin^2 \theta)} \ a \cos \theta \ d\theta$$

$$=\frac{4ba}{\pi |2} \qquad \cos^2 \theta$$

$$=\frac{4ba}{cos^2 \theta} \ d\theta$$
Bands for θ

$$x = 0: \quad 0 = a \sin \theta \rightarrow \sin \theta = 0 \rightarrow \theta = 0$$

$$x = a: \quad a = a \sin \theta \rightarrow \sin \theta = 1 \rightarrow \theta = \pi$$

$$=\frac{4ba}{cos^2 \theta} \ d\theta$$

$$=\frac{4ba}{0} \qquad \text{Vallis' formula} \rightarrow \pi$$

$$=\frac{4ba}{4} = \pi ab$$